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Abstract

TIPS are notes and bonds issued by the U.S. Treasury with coupons and principal payments indexed to inflation. Using no-arbitrage term structure models, we show that TIPS yields contained liquidity premiums as large as 100 basis points when TIPS were first issued, reflecting the newness of the instrument, and up to 350 basis points during the recent financial crisis, reflecting common funding constraints affecting a variety of financial markets. Applying our models to the U.K. data also reveals liquidity premiums in index-linked gilt yields that spiked to nearly 250 basis points at the height of the crisis. Ignoring TIPS liquidity premiums is shown to significantly distort the information content of TIPS yields and TIPS breakeven inflation rate, two widely-used empirical proxies for real rates and expected inflation

1 Introduction

Treasury Inflation-Protected Securities (TIPS) are fixed-income securities whose semiannual coupons and principal payments are indexed to the non-seasonally-adjusted consumption price index for all urban consumers.[1] Since its inception in 1997, the market for TIPS has grown substantially and now comprises about 8% of the outstanding Treasury debt market. More than fifteen years of TIPS data provides a rich source of information to market participants and academic researchers alike. From a portfolio management perspective, TIPS provide a stream of known "real" payments at horizons up to 30 years and are therefore highly attractive to long-term investors such as retirement savings accounts who would like to hedge themselves against inflation risks. In addition, TIPS prices frequently moved in opposite directions of stock prices in recent years and can be used to hedge against equity price variations.[2] More broadly, TIPS yields can be viewed as rough measures of real risk-free interest rates, which are an important determinant of the costs for financing private investment projects and public debt and also provide a better gauge of the stance of monetary policy than nominal interest rates. Information about real yields also has direct implications for asset pricing models, many of which are written in terms of real consumption. Finally, the differential between yields on nominal Treasury securities and on TIPS of comparable maturities, also called the "breakeven inflation rate" (BEI) or "inflation compensation,"is often used as proxy for market participants' inflation expectation, a key variable entering the decisions of firms, investors, and monetary policy makers.

Despite the potential usefulness of such securities, this paper presents evidence that it is essential to take into account the lower liquidity of TIPS relative to their nominal counterparts when using them to measure real interest rates and inflation expectations. In particular, TIPS investors demand premiums for holding these less liquid securities, therefore pushing up TIPS yields above the true real yields and pushing down TIPS BEI below its fundamental levels. Treating the TIPS BEI as a clean proxy for inflation expectation can be especially problematic, since an economi-

[1] Sack and Elsasser (2004) provides a detailed description of the TIPS market.
[2] See, for example, Campbell, Sunderam, and Viceira (2010).

cally significant TIPS liquidity premium combined with a positive inflation risk premium could potentially drive a large wedge between the TIPS BEI and true inflation expectations. We show that the early years of the TIPS market and the recent financial crisis provide two examples when poor liquidity significantly distorted the information content of TIPS prices.

To demonstrate the existence of TIPS liquidity premiums, we model nominal Treasury yields, TIPS yields, and inflation jointly in a no-arbitrage asset pricing framework. We examine several different specifications for such models, where the nominal Treasury-TIPS liquidity differential is either ignored or modeled as an additional stochastic factor that influences TIPS but not nominal Treasury yields. All models we use are from the Gaussian essentially-affine no-arbitrage term structure family and allow flexible specifications for both real and nominal market prices of risk. This approach differs significantly from that of the other two studies of TIPS liquidities in the literature, Shen (2006) and Pflueger and Viceira (2013), who based their results on regression analysis of either TIPS BEI itself or the difference between TIPS BEI and survey-based measures of inflation expectations. The usage of a pricing model allows us to bring in additional information from the cross section of yields and TIPS BEI and realized inflation to better distinguish between the liquidity and the inflation risk premium components of TIPS BEI. Moreover, the Gaussian bond pricing framework used here allows for a flexible correlation structure between the factors, which is important for identifying risk premiums, which arise from correlations between pricing kernels and asset returns. This framework therefore allows an accurate decomposition of the TIPS BEI into expected inflation, the inflation risk premium, and the TIPS liquidity premium.

Our main findings can be summarized as follows. First, a standard 3-factor model that assumes no TIPS liquidity premium generates not only large pricing errors for TIPS yields but also estimates of inflation expectations that differ significantly from survey inflation forecasts and estimates of inflation risk premiums that carry a counterintuitive negative sign. In comparison, the two 4-factor models allowing TIPS yields to differ from the true real yields by a liquidity spread generate notably smaller TIPS pricing errors, more reasonable estimates of inflation risk premiums, and estimated inflation expectations that agree well with survey inflation forecasts. Second, the liquidity premium estimates from both 4-factor models share the feature that they were large ($\sim$1%) in the early years, declined steadily until late 2003, and remained at relatively low levels until the recent crisis, consistent with the notion that TIPS market liquidity conditions had been improving over time. Those estimates jumped to well above 3% in July 2008 during the TIPS sell-off but had largely returned to their pre-crisis levels by the beginning of 2010, a pattern similar to what has been found for many other markets. Third, regression analysis shows that around 77-86% of the variations in our estimates of TIPS liquidity premiums can be explained by observable measures of the liquidity conditions in the TIPS market. Finally, when applied to the UK data, one variant of our model reveals liquidity premiums in index-linked gilt yields that were fairly low in normal times but spiked to about 250 basis points at the height of the recent crisis.

To the best of our knowledge, this is the first paper to model the TIPS liquidity premium in a no-arbitrage framework and as such is related to the fast-growing literature on the link between liquidity and asset returns.[3] Previous studies have documented that assets with similar payoffs can carry significantly different prices due to their liquidity differences.[4] This paper adds to the evidence by showing that although nominal Treasury securities and TIPS are both issued by the U.S. Treasury and perceived as largely free of credit risks, they are priced differently due to their distinct liquidities after controlling for the differences in their real payoffs. Our paper is also related to studies of the pricing of indexed bonds, most of which are applied to countries with longer histories for such bonds than the U.S.,[5] while studies using TIPS and other U.S. inflation-linked assets are fairly recent and relatively few.[6] Most of the studies using TIPS yields take them at their

[3] Vayanos and Wang (2013) and Amihud, Mendelson, and Pedersen (2013) provide recent surveys of this literature.

[4] See, for example, Amihud and Mendelson (1986) and Brennan, Chordia, and Subrahmanyam (1998), and Brennan and Subrahmanyam (1996) for equities, Amihud and Mendelson (1991) and Longstaff (2004) for nominal Treasury securities, Longstaff, Mithal, and Neis (2005), Chen, Lesmond, and Wei (2007), Bao, Pan, and Wang (2011), and Dick-Nielsen, Feldhütter, and Lando (2012) for corporate bonds, Bongaerts, De Jong, and Driessen (2011) for credit default swaps, and Mancini, Ranaldo, and Wrampelmeyer (forthcoming) for the foreign exchange market.

[5] See Woodward (1990), Barr and Campbell (1997), Evans (1998), Remolona, Wickens, and Gong (1998), Risa (2001), and Joyce, Lildholdt, and Sorensen (2010) for the UK, Kandel, Ofer, and Sarig (1996) for Isreal, and Hördahl and Tristani (forthcoming) for the Euro area.

[6] This paper and a contemporaneous study by Chen, Liu, and Cheng (2010) are the first two to study TIPS in a no-arbitrage framework. More recent papers analyzing TIPS or inflation swaps include Chernov and Mueller (2012), Adrian and Wu (2008), Haubrich, Pennacchi, and Ritchken (2012), Christensen, Lopez, and Rudebusch (2010), Pflueger and Viceira (2013), Fleckenstein, Longstaff, and Lustig (Forthcoming), Christensen and Gillan (2011), Fleming and Krishnan (2012), Grishchenko and Huang (2013), Abrahams, Adrian, Crump, and Moench (2013).

face value and as a result typically produce real yield estimates that seem too high and inflation risk premium estimates that are insignificant or negative in the few years following the introduction of TIPS. In contrast, this paper shows that there is a persistent liquidity premium component in TIPS prices that, when ignored, will significantly bias the results. Finally, this paper is also related to the vast literature studying the behavior of real interest rates, inflation expectations, and inflation risk premiums but without incorporating information from indexed bonds.[7]

The remainder of this paper is organized as follows. In Section 2, we provide evidence that TIPS yields and TIPS breakeven inflation contain an additional factor, likely reflecting the relative illiquidity of TIPS, beyond those driving the nominal interest rates. Section 3 spells out the details of the no-arbitrage models we use, including the specification of the additional liquidity factor. Section 4 describes the data and our empirical methodology, and Section 5 presents the main empirical results. Section 6 provides further discussions on the model estimates of the TIPS liquidity premiums, showing that those estimates are indeed linked to the liquidity conditions in the TIPS markets and that they account for a significant portion of the time series variations in TIPS BEI. Section 7 applies the model to the U.K. data. Finally, Section 8 concludes.

2 A TIPS Liquidity Factor: Simple Analysis

In this section we present evidence that there is a component of TIPS yields that is not reflected in nominal Treasury yields but is related to the liquidity of the TIPS market. This serves as the motivation for introducing a TIPS-specific factor when we model nominal and TIPS yields jointly in later sections.

Simple Regression Analysis

In the first analysis, we regress the 10-year TIPS BEI, defined as the spread between the 10-year nominal yield and the 10-year TIPS yield, on 3-month, 2-year and 10-year nominal yields as

[7] See, e.g., Pennacchi (1991), Foresi, Penati, and Pennacchi (1997), and Brennan, Wang, and Xia (2004), Buraschi and Jiltsov (2005), and Ang, Bekaert, and Wei (2008).

well as a constant:[8]

$$BEI_{t,10}^{\mathcal{T}} = \alpha + \beta_1 y_{t,0.25}^N + \beta_2 y_{t,2}^N + \beta_3 y_{t,10}^N + e_t. \tag{1}$$

Standard finance theory suggests that nominal yield of any maturity can be decomposed into the underlying real yield, inflation expectation and the inflation risk premium:

$$y_{t,\tau}^N = y_{t,\tau}^R + I_{t,\tau} + \wp_{t,\tau}, \tag{2}$$

where $y_{t,\tau}^N$ and $y_{t,\tau}^R$ are the τ-period nominal and real yields, respectively, $I_{t,\tau}$ is the expected inflation over the next τ periods and $\wp_{t,\tau}$ is the inflation risk premium. If TIPS yields are a good measure of the underlying real yields, the TIPS breakeven inflation rate is simply the sum of expected inflation and the inflation risk premium, which are also parts of the nominal yields. In that case, a regression of TIPS BEI onto nominal yield curve factors can be expected to result in a high R^2. On the other hand, variations in TIPS yields that are unrelated to those in the underlying real yields could lead to a low R^2.

Regression (1) is estimated both in levels and in weekly differences using three samples. The first sample corresponds to the full sample period of January 6, 1999 to March 27, 2013, which is then split into two sub-samples: the pre-crisis period from January 6, 1999 to December 31, 2007, and the post-crisis period from January 2, 2008 to March 27, 2013. The results are reported in the left columns in Panel A of Table 1. The R^2 from the full-sample regression is merely 6% in levels and 39% in weekly differences, well below the levels of R^2 in excess of 95% typically reported when regressing one nominal yield onto other nominal yields. In the pre-crisis period, the R^2 is higher for both specifications, 30% in levels and 57% in differences, indicating that the very low R^2 is mostly due to the post-crisis period, during which it declines to 5% and 27% in levels and differences, respectively. This evidence suggests that a large portion of variations in the 10-year BEI cannot be explained by factors driving the nominal yields, and even more so when the most

[8] We thank Greg Duffee for this suggestion. Results using three different nominal yields or using the first principal components of nominal yields are similar. 3-month T-bill yields are from the Federal Reserve Board's H.15 release, while longer-term nominal yields and the 10-year TIPS yield are from fitted yield curves maintained by the staff at the Federal Reserve Board of Governors (for more details, see Section 4.1).

recent financial crisis is included in the sample.

Principal Components Analysis

Next, we conduct a principal component analysis (PCA) of the cross section of nominal and TIPS yields over the full sample. It is well known that, in the case of nominal yields, three factors explain most of the nominal yield curve movements. This is confirmed by looking at the left panels in Table 1, Panel B: Over 97% of variations in the weekly changes of nominal yields can be explained by the first three principal components; once we add TIPS yields, at least four factors are needed to explain the same portion of the total variance. The left columns in Panel C of Table 1 report the correlations between the first four PCA factors extracted from nominal yields alone and those from nominal and TIPS yields jointly. It is interesting to note that, the first, second, and fourth factors constructed from all yields largely retain their interpretations as the level, slope and curvature of the nominal yield curve, as can be seen from their high correlations with the first, the second and the third nominal factors, respectively. However, the third PCA factor extracted from nominal and TIPS yields combined is not highly correlated with any of the nominal PCA factors.

A Case for TIPS Liquidity Premium

One interpretation of the TIPS-specific factor we mentioned above is that it reflects a "liquidity premium": investors would demand a compensation for holding a relative new and illiquid instrument like TIPS, either in the early years or during episodes like the most recent financial crisis.

Indeed, several measures related to TIPS market liquidity conditions, as well as anecdotal reports, indicate that the liquidity in TIPS market was much poorer than that of nominal securities, and that TIPS market liquidity improved over time, although this improvement was not a smooth, steady process. The top panel of Figure 1 shows the gross TIPS issuance over the period 1997-2013. The TIPS issuance dipped slightly in 2000-2001 before rising substantially in 2004. According to Sack and Elsasser (2004), there were talks around 2001 that the Treasury might discontinue the TIPS due to the relatively weak demand for TIPS. TIPS outstanding, shown in the bottom panel of Figure 1, began to grow at a faster pace from 2004 onward and now exceeds 800 billion dollars. Figure 2 tells a similar story from the demand side: TIPS transaction volumes grew almost tenfold between 1999 and 2004, and TIPS mutual funds also experienced significant growth.[9] . Transaction volumes declined sharply at the end of 2008 and remained at a lower level until 2010. In view of this institutional history, it is not unreasonable to suppose that TIPS contained a significant liquidity premium in its early years and again during the financial crisis.

[Insert Figure 1 about here.]

[Insert Figure 2 about here.]

A liquidity premium in TIPS can also help resolve an apparent inconsistency between long-term survey inflation forecasts—proxied by the 10-year inflation forecasts from the Survey of Professional Forecasters (SPF) and the long-term inflation forecast from the Thompson Reuters/Michigan Survey of Consumers—and the 10-year TIPS BEI, all of which are plotted in Figure 3.[10] Recall that the true BEI, defined as the yield differential between nominal and real bonds of comparable maturities, is the sum of expected inflation and the inflation risk premium, therefore would tend to be higher than inflation expectations from surveys, and can be considered as a good measure of true expected inflation if the inflation risk premium is relatively small and does not vary too much over time. However, Figure 3 shows that this is not the case: the TIPS BEI lied below both measures of survey inflation forecasts almost all the time before 2004.[11] [12] In addition, TIPS yields surged shortly after Lehman failed while conventional nominal yields plummeted, pushing 10-year TIPS BEI close to zero at the end of 2008, while survey inflation expectations were practically unchanged over the same period. Such disparities cannot be attributed solely to the existence

[9] Data on TIPS mutual fund is from the Investment Company Institute.

[10] Keane and Runkle (1990) show that SPF forecasts of the GNP deflator are both unbiased and efficient. Mehra (2002) shows that Michigan survey forecasts of inflation are unbiased, efficient, and have predictive content for future inflation. Ang, Bekaert, and Wei (2007) and Chun (2012) show that SPF, Blue Chip, and other surveys forecast inflation better than many types of models estimated with yields only. Finally, using a macro-finance term structure model, Chernov and Mueller (2012) show that forecasts of inflation from all above-mentioned surveys are consistent with inflation expectations embedded in yields.

[11] This result is not specific to the use of survey inflation as a proxy for inflation expectations. Other measures of inflation expectation based on time-series models also tend to be above the TIPS in early years.

[12] Similar points are made by Shen and Corning (2001) and Shen (2006).

of inflation risk premium, as such an explanation would require the inflation risk premium to be mostly negative during these two episodes and highly volatile, which stands in contrast with most studies in the literature that find inflation risk premiums to be positive on average and relatively smooth.[13]

A positive TIPS liquidity premium, on the other hand, would push the TIPS-based BEI below the true BEI and potentially even below survey inflation forecasts, if the TIPS liquidity premium exceeds the inflation risk premium. Part of the volatility of the TIPS BEI may also be due to the volatility of the TIPS liquidity premium. In order to study these issues quantitatively, we need a framework for identifying and measuring the relevant components, including the TIPS liquidity premium, inflation expectations, and inflation risk premium. For this purpose, we use the no-arbitrage term structure modeling framework, to which we now turn.

[Insert Table 1 about here.]

[Insert Figure 3 about here.]

3 A Joint Model of Nominal Yields, TIPS yields, and Inflation

This section details the no-arbitrage framework that we use to model nominal and TIPS yields and inflation jointly. The no-arbitrage approach has the benefit of avoiding the tight assumptions that go into structural, utility-based models, while still allowing term structure variations to be modeled in a dynamically consistent manner by requiring the cross section of yields to satisfy the no-arbitrage restrictions.

[13] See Campbell and Shiller (1996), Foresi, Penati, and Pennacchi (1997), Veronesi and Yared (1999), Buraschi and Jiltsov (2005), Ang, Bekaert, and Wei (2008), among others. Hördahl and Tristani (2010) provides a nice overview of some recent development.

3.1 State Variable Dynamics and the Nominal Pricing Kernel

We assume that real yields, expected inflation, and nominal yields are all driven by a vector of three latent variables, $x_t = [x_{1t}, x_{2t}, x_{3t}]'$, that follows a multivariate Gaussian process,

$$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t, \tag{3}$$

where B_t is an 3×1 vector of standard Brownian motion, μ is a 3×1 constant vector, and $\mathcal{K}, \Sigma$ are 3×3 constant matrices.

The nominal short rate is specified as

$$r^N(x_t) = \rho_0^N + \rho_1^{N\prime} x_t, \tag{4}$$

where ρ_0^N is a constant and ρ_1^N is a 3×1 vector.

The nominal pricing kernel takes the form

$$dM_t^N / M_t^N = -r^N(x_t)dt - \lambda^N(x_t)' dB_t, \tag{5}$$

where the vector of nominal prices of risk is given by

$$\lambda^N(x_t) = \lambda_0^N + \Lambda^N x_t, \tag{6}$$

in which λ_0^N is a 3×1 vector and Λ^N is a 3×3 matrix. Note that the nominal term structure in this paper is fairly standard and falls into the "essentially affine" $A_0(3)$ category developed by Duffee (2002).

3.2 Inflation and the Real Pricing Kernel

The price level processes takes the form:

$$d \log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp}. \tag{7}$$

where the instantaneous expected inflation, $\pi(x_t)$, is an affine function of the state variables in the form of

$$\pi(x_t) = \rho_0^{\pi} + \rho_1^{\pi'} x_t, \tag{8}$$

and the unexpected inflation, $\sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp}$, is allowed to load on shocks that move the nominal interest rates and expected inflation, dB_t, and on an orthogonal shock $dB_t^{\perp}$ with $dB_t dB_t^{\perp} = 0_{3\times 1}$. The orthogonal shock is included to capture short-run inflation variations that may not be spanned by yield curve movements.

A real bond can be thought of as a nominal asset paying realized inflation upon maturity. Therefore, the real and the nominal pricing kernels are linked by the no-arbitrage relation

$$M_t^R = M_t^N Q_t. \tag{9}$$

Applying Ito's lemma to Equation (9) and using Equations (4) to (8), the real pricing kernel can be derived as following the process

$$dM_t^R / M_t^R = dM_t^N / M_t^N + dQ_t / Q_t + (dM_t^N / M_t^N) \cdot (dQ_t / Q_t) \tag{10}$$

$$= -r^R(x_t)dt - \lambda^R(x_t)' dB_t - (\cdot) dB_t^{\perp} \tag{11}$$

where the real short rate is given by

$$r^R(x_t) = \rho_0^R + \rho_1^{R'} x_t, \tag{12}$$

the vector of real prices of risk is given by

$$\lambda^R(x_t) = \lambda_0^R + \Lambda^R x_t, \tag{13}$$

and the coefficients are linked to their nominal counterparts by

$$\rho_0^R = \rho_0^N - \rho_0^\pi - \frac{1}{2}(\sigma_q'\sigma_q + \sigma_q^{\perp 2}) + \lambda_0^{N'}\sigma_q \tag{14}$$

$$\rho_1^R = \rho_1^N - \rho_1^\pi + \Lambda^{N'}\sigma_q \tag{15}$$

$$\lambda_0^R = \lambda_0^N - \sigma_q \tag{16}$$

$$\Lambda^R = \Lambda^N. \tag{17}$$

3.3 Nominal and Real Bond Prices

By the definition of nominal and real pricing kernels, the time-t prices of τ-period nominal and real bonds, $P_{t,\tau}^N$ and $P_{t,\tau}^R$, are given by

$$P_{t,\tau}^i = E_t(M_{t+\tau}^i)/M_t^i, \quad i = N, R. \tag{18}$$

The bond prices can be also expressed in terms of a risk-neutral expectation as

$$P_{t,\tau}^i = E_t^Q \left(\exp\left(-\int_t^{t+\tau} r_s^i \, ds \right) \right), \quad i = N, R. \tag{19}$$

where the superscript Q denotes the risk-neutral measure.

Following the standard literature,[14] it is straightforward to derive a closed-form solution for the bond prices:

$$P_{t,\tau}^i = \exp\left(A_\tau^i + B_\tau^{i\prime} x_t \right), \quad i = N, R, \tag{20}$$

[14] See Duffie and Kan (1996) and Dai and Singleton (2000), among others.

where

$$\frac{dA^i_\tau}{d\tau} = -\rho^i_0 + B^{i\prime}_\tau \left(\mathcal{K}\mu - \Sigma\lambda^i_0\right) + \frac{1}{2}B^{i\prime}_\tau \Sigma\Sigma' B^i_\tau \tag{21}$$

$$\frac{dB^i_\tau}{d\tau} = -\rho^i_1 - \left(\mathcal{K} + \Sigma\Lambda^i\right)' B^i_\tau \tag{22}$$

with initial conditions $A^i_0 = 0$ and $B^i_0 = 0_{3\times 1}$.

Nominal and real yields therefore both take the affine form,

$$y^i_{t,\tau} = a^i_\tau + b^{i\prime}_\tau x_t, \quad i = N, R, \tag{23}$$

where the factor loadings a^i and b^i are given by

$$a^i_\tau = -A^i_\tau/\tau, \;\; b^i_\tau = -B^i_\tau/\tau, \tag{24}$$

3.4 Inflation Expectations and Inflation Risk Premiums

In this model, inflation expectations also take an affine form,

$$I_{t,\tau} \triangleq E_t(\log(Q_{t+\tau}/Q_t))/\tau = a^I_\tau + b^{I\prime}_\tau x_t, \tag{25}$$

where the factor loadings a^I and b^I are given by

$$a^I_\tau = \rho^\pi_0 + (1/\tau)\rho^{\pi\prime}_1 \int_0^\tau ds(I - e^{-\mathcal{K}s})\mu$$

$$b^I_\tau = (1/\tau)\int_0^\tau ds\, e^{-\mathcal{K}'s}\rho^\pi_1,$$

From equations (23)-(25), it can be seen that both the BEI, defined as the difference between zero coupon nominal and real yields of identical maturities, and the inflation risk premium, defined as the difference between the BEI and the expected log inflation over the same horizon, are affine in the state variables:

$$BEI_{t,\tau} \triangleq y^N_{t,\tau} - y^R_{t,\tau} = a^N_\tau - a^R_\tau + (b^N_\tau - b^R_\tau)' x_t. \tag{26}$$

and

$$\wp_{t,\tau} \triangleq y^N_{t,\tau} - y^R_{t,\tau} - I_{t,\tau} = a^N_\tau - a^R_\tau - a^I_\tau + (b^N_\tau - b^R_\tau - b^I_\tau)' x_t. \tag{27}$$

Using Equation (9) we can write the price of a τ-period nominal bond as

$$P^N_{t,\tau} = E_t(M^R_{t+\tau} Q^{-1}_{t+\tau})/(M^R_t Q^{-1}_t). \tag{28}$$

It is then straightforward to show that the inflation risk premium $\wp_{t,\tau}$ consists of a covariance term, $c_{t,\tau}$, and a Jensen's inequality term, $J_{t,\tau}$:

$$\wp_{t,\tau} = c_{t,\tau} + J_{t,\tau}, \tag{29}$$

where

$$\begin{aligned} c_{t,\tau} &\equiv -(1/\tau)\log[1 + cov_t(M^R_{t+\tau}/M^R_t, Q_t/Q_{t+\tau})/(E_t(M^R_{t+\tau}/M^R_t)E_t(Q_t/Q_{t+\tau}))] \\ J_{t,\tau} &\equiv -(1/\tau)[\log(E_t(Q_t/Q_{t+\tau})) - E_t(\log(Q_t/Q_{t+\tau}))]. \end{aligned}$$

In practice, the Jensen's inequality term is fairly small, and the inflation risk premium is mainly determined by the covariance between the real pricing kernel and inflation, and can assume either a positive or a negative sign depending on how the two terms covaries over time.

3.5 A Four-Factor Model of TIPS Yields

Given the evidence presented in Section 2 on the existence of a TIPS-specific factor, we allow the TIPS yield to deviate from the true underlying real yield. The spread between the TIPS yield and the true real yield,

$$L_{t,\tau} = y^{\mathcal{T}}_{t,\tau} - y^{R}_{t,\tau}, \tag{30}$$

primarily captures the liquidity premium TIPS investors demand for holding an instrument that is less liquid than nominal Treasury securities, but may also reflect other factors that can potentially drive a wedge between the TIPS yield and the true real yield.[15] Since the relative illiquidity of TIPS would lower TIPS prices and raise TIPS yields, we would in general expect $L_{t,\tau}$ to be positive.

To model $L_{t,\tau}$, we assume that the investors discount TIPS cash flows by adjusting the true instantaneous real short rate with a positive liquidity spread, resulting in a TIPS yield that exceeds the true real yield by

$$L_{t,\tau} = -(1/\tau)\log E^{Q}_{t}\left(\exp\left(-\int_{t}^{t+\tau}(r^{R}_{s} + l_{s})ds\right)\right) - y^{R}_{t}, \tag{31}$$

where l_t is the instantaneous liquidity spread, and the superscript Q represents expectation taken under the risk-neutral measure. This is analogous to the corporate bond pricing literature, where defaultable bonds are priced by discounting future cash flows using a default- and liquidity-adjusted short rate.[16] Note that, without the instantaneous liquidity spread l in Equation (31), the TIPS yield becomes identical to the true real yield y^R and the TIPS liquidity premium becomes zero (see Equation (19)).

[15] Such factors include indexation lags, seasonal and short-run variations in headline CPI prices, and the embedded deflation option in newly issued TIPS (e.g. Grishchenko, Vanden, and Zhang (2011)). We note though that the TIPS yields used in this study appear to be little affected by the embedded deflation options, as only a few outstanding TIPS had deflation options that were in the money and the fitted yield curve was essentially unchanged when those TIPS were dropped from the curve estimation.

[16] See Duffie and Singleton (1999), Longstaff, Mithal, and Neis (2005), Driessen (2005).

The instantaneous liquidity spread l_t is given by

$$l_t = \gamma' x_t + \tilde{\gamma}\tilde{x}_t, \tag{32}$$

where γ is a 3×1 constant vector, $\tilde{\gamma}$ is a constant, and $\tilde{x}_t$ follows the Vasicek (1977) process and is independent of all other state variables contained in x_t:

$$d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma} dW_t, \tag{33}$$

in which $dW_t dB_t = 0_{3\times 1}$. A non-zero γ allows the liquidity premium to be correlated with the state of the economy. We assume that the independent liquidity factor $\tilde{x}_t$ carries a market price of risk of

$$\tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{x}_t. \tag{34}$$

Appendix A shows that the TIPS liquidity premium takes the affine form

$$L_{t,\tau} = \left[\tilde{a}_\tau + (a_\tau^{\mathcal{T}} - a_\tau^R)\right] + \left[\begin{array}{cc} (b_\tau^{\mathcal{T}} - b_\tau^R)' & \tilde{b}_\tau \end{array}\right] \left[\begin{array}{c} x_t \\ \tilde{x}_t \end{array}\right] \tag{35}$$

Note that although we focus on a one-factor specification for the liquidity factor $\tilde{x}_t$, it is straightforward to extend the model to incorporate multiple liquidity factors.

The TIPS yields in our model is given by

$$y_{t,\tau}^{\mathcal{T}} = y_{t,\tau}^R + L_{t,\tau}. \tag{36}$$

We shall refer to this model as Model L-II. In the empirical part, we also estimate two restricted versions of the full model. The first restricted model, which we shall term Model L-I, sets $\gamma = 0_{3\times 1}$ in Equation (32), so that the liquidity premiums on TIPS are not correlated with the nominal term structure factors. This results in a model similar to those in Driessen (2005) and Longstaff, Mithal, and Neis (2005), which model the liquidity premium in corporate bonds as a one-factor process that is independent of the credit and interest rate factors. The second restricted model, which we shall call Model NL, simply equate TIPS yields with the true real yields, i.e.,

$$y^{\mathcal{T}}_{t,\tau} = y^{R}_{t,\tau}. \tag{37}$$

This is the model studied by Chen, Liu, and Cheng (2010), although their specification differs from ours along other dimensions.[17]

3.6 A Comparison to Previous Studies

Besides its tractability, the affine-Gaussian bond pricing framework used here allows for a flexible correlation structure between the factors. As the inflation risk premium arises from the correlation between the real pricing kernel and inflation, it is important to allow for a general correlation structure. On the other hand, the affine-Gaussian setup does not capture the *time-varying* inflation uncertainty and therefore cannot further decompose inflation risk premiums into the part due to time-varying inflation risks and time-varying prices of inflation risk. Nonetheless, the affine-Gaussian model could still provide reasonable estimates of the inflation risk premium, similar to the way it generates reasonable measures of term premia despite its inability to capture time-varying interest rate volatilities. We view the general affine-Gaussian model as an important benchmark to investigate before exploring more sophisticated models.

Some of the models studied in the earlier literature, such as Pennacchi (1991) and Campbell and Viceira (2001), can be viewed as special cases of this model. For example, Pennacchi (1991)'s model corresponds to a two-factor version of our model with constant market price of risk. Campbell and Viceira (2001) is also a special case of this model, as their real term structure has a lower dimension than the nominal term structure, with nominal yields described by two factors and real

[17] Chen, Liu, and Cheng (2010) treat TIPS yields as true real yields and ignore any potential liquidity premiums. Their model is of the CIR type, which is known to have problems in fitting risk premiums. The instantaneous inflation expectation in their model follows a non-negative process, which precludes the possibility of deflation.

yields described by one. In this paper, we let the data decide whether the real term structure should have the same dimensionality as the nominal term structure or a lower one. A related point is that in a reduced-form setup like ours, one cannot classify factors into *real* and *nominal* ones, as the correlation effects in the model make such a distinction meaningless.

Overall, compared with previous studies, two main features of this model help us better distinguish the inflation risk premium and the liquidity premium components of the TIPS BEI. On the one hand, the use of price level data Q_t in the estimation and the unrestricted correlation structure between factor innovations help us better pin down expected inflation and the inflation risk premium. On the other hand, the higher-dimensionality of the real term structure, the estimation of which is assisted by the additional information from TIPS yields, allows us to better identify the parameters governing the "true" real yields dynamic. As a result, the wedge between TIPS and true real yields, our measure of liquidity premium, is pinned down, and can be estimated separately from the inflation risk premium. These features cannot be fully appreciated unless considered within the context of the empirical methodology used to estimate the model, which is described in the next section.

4 Data and Empirical Methodology

4.1 Data

We use 3- and 6-month, 1-, 2-, 4-, 7-, and 10-year nominal yields and CPI-U data from January 1990 to March 2013. In contrast, our TIPS yields are restricted by data availability and cover a shorter period from January 1999 to March 2013, with the earlier period without TIPS data (1990-1998) treated as missing observations.[18] We sample yields at the weekly frequency and

[18] 3- and 6-month T-bill yields are from the Federal Reserve Board's H.15 release and converted to continuously compounded basis. Longer-term nominal yields and TIPS yields are based on zero-coupon yield curves fitted at the Federal Reserve Board. In particular, nominal yields are based on the Svensson (1995) curve specification for the entire sample; TIPS yields are based on the Nelson and Siegel (1987) curve specification prior to January 2004 and the Svensson (1995) curve specification thereafter. See Gürkaynak, Sack, and Wright (2007, 2010) for details.

assume that the monthly CPI-U data is observed on the last Wednesday of the current month.[19] As discussed in the internet appendix, shorter-maturity TIPS yields are affected to a larger degree by the problem of indexation lags. They also cannot be estimated reliably before 2002, as only one TIPS with maturities below 5 years existed over that period. We therefore only use 5-, 7-, and 10-year TIPS yields in our estimation. All nominal and TIPS yields used in the estimation are plotted in Figure 4. TIPS are indexed to non-seasonally-adjusted CPI; however,because the models we estimate do not accommodate seasonalities, we use seasonally-adjusted CPI inflation in the estimation. This is not expected to have a big effect due to the relatively long maturities of TIPS yields that we examine.

[Insert Figure 4 about here.]

The sample period 1990-2013 was chosen as a compromise between having more data in order to improve the efficiency of estimation, and having a more homogeneous sample so as to avoid possible structural breaks[20] in the relation between term structure variables and inflation. This sample period roughly spans Greenspan's tenure and most of Bernanke's as well.

Results from Kim and Orphanides (2012) suggest that the standard technique of estimating our models using only nominal and TIPS yields and inflation data for a relative short sample period of 1990-2013 will almost surely run into small sample problems: variables like $\mathcal{K}$ and Λ^N may not be reliably estimated, and the model-implied path of expected future short rate over the next 5 to 10 years is typically too smooth compared to survey-based measures of interest rates expectations. Therefore, we supplement the aforementioned data with survey forecasts of 3-month T-bill yields to help stabilize the estimation and to better pin down some of the model parameters. Note that survey *inflation* forecast data are *not* used in the estimation, as we would like to use survey inflation forecasts as a means for model testing.

To be specific, we use the 6- and 12-month-ahead forecasts of the 3-month T-bill yield from Blue Chip Financial Forecasts, which are available monthly, and allow the size of the measurement

[19] Here we abstract from the real-time data issue by assuming that investors correctly infer the current inflation rate in a timely fashion.

[20] The 1979-83 episode of Fed's experiment with reserve targeting is a well known example.

errors to be determined within the estimation. We also use long-range forecast of 3-month T-bill yield over the next 5 to 10 years from the same survey, which are available twice a year, with the standard deviation of its measurement error fixed at a fairly large value of 75 basis points at an annual rate. This is done to prevent the long-horizon survey forecasts from having unduly strong influence on the estimation, and can be viewed as similar to a quasi-informative prior in a Bayesian estimation.

We denote the short-horizon survey forecasts by $f_{t,6m}$ and $f_{t,12m}$ and the long-range forecast by $f_{t,long}$. The standard deviation of their measurement errors are denoted denoted $\delta_{f,6m}, \delta_{f,12m}$ and $\delta_{f,long}$, respectively. These survey-based forecasts are taken as noisy measures of the corresponding true market expectations. More specifically, we assume that the short-term survey forecasts take the form of

$$f_{t,\tau} = E_t(y^N_{t+\tau,3m}) + \epsilon^f_{t,\tau}, \qquad \epsilon^f_{t,\tau} \sim N(0, \delta^2_{f,\tau}), \tag{38}$$

with $\tau = 6m$ or $12m$, and the long-range forecasts take the form of

$$f_{t,long} = E_t\left(\frac{1}{5}\int_{5y}^{10y} y^N_{t+\tau,3m} d\tau\right) + \epsilon^f_{t,long}, \qquad \epsilon^f_{t,long} \sim N(0, 0.0075^2). \tag{39}$$

The corresponding model forecasts, $E_t(y^N_{t+\tau,3m})$, can be solved from Equations (3) and (23) and can be shown to be

$$E_t(y^N_{t+\tau,3m}) = a^f_\tau + b^{f\prime}_\tau x_t, \tag{40}$$

with the factor loadings a^f_τ, b^f_τ given by

$$a^f_\tau = a^N_{3m} + b^{N\prime}_{3m}\left(I_{3\times 3} - e^{-\mathcal{K}\tau}\right)\mu \tag{41}$$

$$b^f_\tau = e^{-\mathcal{K}'\tau} b^N_{3m} \tag{42}$$

4.2 Identification and Summary of Models

We only impose restrictions that are necessary for achieving identification so as to allow a maximally flexible correlation structure between the factors, which has shown to be critical in fitting the rich behavior in risk premiums that we observe in the data.[21] In particular, we employ the following normalization:

$$\mu = 0_{3\times 1}, \ \ \Sigma = \begin{bmatrix} 0.01 & 0 & 0 \\ \Sigma_{21} & 0.01 & 0 \\ \Sigma_{31} & \Sigma_{32} & 0.01 \end{bmatrix}, \ \ \mathcal{K} = \begin{bmatrix} \mathcal{K}_{11} & 0 & 0 \\ 0 & \mathcal{K}_{22} & 0 \\ 0 & 0 & \mathcal{K}_{33} \end{bmatrix}, \ \ \tilde{\sigma} = 0.01. \tag{43}$$

and leave all other parameters unrestricted. It can be shown that any specification of the affine Gaussian model that has a $\mathcal{K}$ matrix with all-real eigenvalues can be transformed to this form.[22]

To summarize, we estimate three models that differ in how TIPS liquidity premium is modeled, including one model that equates TIPS yields with true real yields and assumes zero liquidity premium on TIPS (Model NL), a model with an independent liquidity factor (Model L-I), and a model allowing the TIPS liquidity premium to be correlated with other state variables (Model L-II). Table 2 summarizes the models and the parameter restrictions. Two things are worth noting here: First, as shown in Section 3.5, Models NL and L-I can all both considered as special cases of Model L-II. Second, Model NL has a 3-factor representation of TIPS yields, while in the other two models TIPS yields have a 4-factor specification.

[Insert Table 2 about here.]

[21] See Duffie and Kan (1996) and Dai and Singleton (2000).

[22] With normalization (43), the specification we estimate in this paper can be shown to be equivalent to that of Sangvinatsos and Wachter (2005). The main difference between their paper and ours is empirical: they use a much longer sample, which would be desirable if the relationship between inflation and interest rates can be assumed to be stable.

4.3 Estimation Methodology

We rewrite the model in a state-space form and estimate it by the Kalman filter. Denote by $\mathrm{x}_t = [q_t, x_t', \tilde{x}_t]'$ the augmented state vector including the log price level, $q_t \equiv \log(Q_t)$, and the TIPS liquidity factor, $\tilde{x}_t$. The state equation is derived as Euler discretization of equations (3), (7), and (33) and can be written in a matrix form as

$$\mathrm{x_t = G_h + \Gamma_h x_{t-h} + \eta_t^x}. \tag{44}$$

where

$$G_h = \begin{bmatrix} \rho_0^\pi h \\ \mathcal{K}\mu h \\ \tilde{\kappa}\tilde{\mu}h \end{bmatrix}, \ \Gamma_h = \begin{bmatrix} 1 & \rho_1^{\pi\prime} h & 0 \\ 0 & I_{3\times 3} - \mathcal{K}h & 0 \\ 0 & 0 & 1 - \tilde{\kappa}h \end{bmatrix} \text{ and } \eta_t^{\mathrm{x}} = \begin{bmatrix} \sigma_q'\eta_t + \sigma_q^\perp \eta_t^\perp \\ \Sigma\eta_t \\ \tilde{\sigma}\tilde{\eta}_t \end{bmatrix}$$

in which η_t, $\eta_t^\perp$, and $\tilde{\eta}_t$ are independent of each other, and have the distribution

$$\eta_t \sim N(0, hI_{n\times n}), \ \ \eta_t^\perp \sim N(0,h), \ \ \tilde{\eta}_t \sim N(0,h). \tag{45}$$

We specify the set of nominal yields as $Y_t^N = \{y_{t,\tau_i}^N\}_{i=1}^7$, and the set of TIPS yields as $Y_t^{\mathcal{T}} = \{y_{t,\tau_i}^{\mathcal{T}}\}_{i=1}^3$, and collect in y_t all data used in the estimation, including the log price level q_t, all nominal yields Y_t^N, all TIPS yields $Y_t^{\mathcal{T}}$, and 6 month-ahead, 12 month-ahead, and long-horizon forecasts of future 3-month nominal yield:

$$\mathrm{y}_t = [q_t, Y_t^N, Y_t^{\mathcal{T}}, f_{t,6m}, f_{t,12m}, f_{t,long}]'. \tag{46}$$

We assume that all nominal and TIPS yields and survey forecasts of nominal short rate are observed with error. The observation equation therefore takes the form

$$\mathrm{y}_t = A + B\mathrm{x}_t + \mathrm{e}_t \tag{47}$$

where

$$A = \begin{bmatrix} 0 \\ A^N \\ \tilde{a} + A^{\mathcal{T}} \\ a^f_{6m} \\ a^f_{12m} \\ a^f_{long} \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & B^{N\prime} & 0 \\ 0 & B^{\mathcal{T}\prime} & \tilde{b}^\prime \\ 0 & b^{f\prime}_{6m} & 0 \\ 0 & b^{f\prime}_{12m} & 0 \\ 0 & b^{f\prime}_{long} & 0 \end{bmatrix}, \quad \mathrm{e}_t = \begin{bmatrix} 0 \\ e^N_t \\ e^{\mathcal{T}}_t \\ e^f_{t,6m} \\ e^f_{t,12m} \\ e^f_{t,long} \end{bmatrix}, \tag{48}$$

in which A^i and B^i, $i = N, \mathcal{T}$ collect the nominal and TIPS yield loadings on x_t, respectively, in obvious ways, and $\tilde{a}$ and $\tilde{b}$ collects the TIPS yield loadings on the independent liquidity factor $\tilde{x}_t$. We assume a simple i.i.d. structure for the measurement errors so that

$$e^N_{t,\tau_i} \sim N(0, \delta^2_{N,\tau_i}), \quad e^{\mathcal{T}}_{t,\tau_i} \sim N(0, \delta^2_{\mathcal{T},\tau^\prime_i}), \quad e^f_{t,\tau_i} \sim N(0, \delta^2_{f,\tau_i}) \tag{49}$$

Based on the state equation (44) and the observation equation (47), it is straightforward to implement the Kalman-filter and perform the maximum likelihood estimation. The details are given in Appendix B. Two aspects are worth noting here: first, the log price level q_t is nonstationary, so we use a diffuse prior for q_t when initializing the Kalman filter. Second, inflation, survey forecasts, and TIPS yields are not available for all dates, which introduces missing data in the observation equation and are handled in the standard way by allowing the dimensions of the matrices A and B in Equation (47) to be time-dependent (see, for example, Harvey (1989, sec. 3.4.7)).

To facilitate the estimation and also to make the results easily replicable, we follow the following steps in estimating all our models:

1. We first perform a "pre"-estimation where a set of preliminary estimates of the parameters governing the nominal term structure is obtained using nominal yields and survey forecasts of 3-month T-bill yield alone.
2. Based on these estimates and data on nominal yields, we can obtain a preliminary estimate of the state variables, x_t.

3. A regression of the monthly inflation onto estimates of x_t obtained in the second step gives a preliminary set of estimates of the parameters governing the inflation dynamics.
4. Finally, these preliminary estimates are used as starting values in the full, one-step estimation of all model parameters.

5 Empirical Results

In this Section, we discuss and compare the empirical performance of the various Models. As we shall see, there are notable differences between the model equating TIPS yields with the true real yields (Model NL) and the models that allow the two sets of yields to differ by a liquidity premium component (Models L-I and L-II).

5.1 Parameter Estimates and Overall Fit

Parameter Estimates

Table 3 reports selected parameter estimates for all three models.[23] Four things are worth noting here: First, estimates of parameters governing the nominal term structure seem to be fairly robustly estimated and are almost identical across all models. In particular, all estimations uncover a factor that is fairly persistent with a half life of about 5 years. All three models also exhibit a similar fit to nominal yields and survey forecasts of nominal short-term interest rates, generating fitting errors in the order of 6 basis points or less for most maturities and slightly larger at around 13 basis points for the 3-month yield.

Second, there are notable differences in the estimates of parameters governing the expected inflation process. In particular, the loading of the instantaneous inflation on the second and the most persistent factor, $\rho^{\pi}_{1,2}$, is negligible in the model without a TIPS liquidity factor (Model NL) but becomes positive and more significant in the two models with a TIPS liquidity factor (Models L-I and L-II). As a result, the monthly autocorrelation of one-year-ahead inflation expectation is

[23] Complete parameter estimates can be found in the internet appendix.

about 0.9 in Model NL but above 0.99 in all other models. As we will see later, the lack of persistence in the inflation expectation process prevents Model NL from generating meaningful variations in longer-term inflation expectations as we observe in the data.

Third, parameter estimates for the TIPS liquidity factor process are significant in both Models L-I and L-II and assume similar values. The price of liquidity risk depends negatively on the liquidity factor, as can been from the negative $\tilde{\lambda}_1$, implying that the same amount of liquidity risk carries higher risk premiums when liquidity is poor. This is intuitive as one would generally expect any deterioration of liquidity conditions to occur during bad economic times. In Model L-II, the loading of the instantaneous TIPS liquidity premium on each of the three state variables, γ, is estimated to be indistinguishable from zero; however, a likelihood ratio test shows that they are jointly significant.

Finally, the fit to TIPS yields is significantly better in models with a TIPS liquidity factor. For example, the fitting errors on the 10-year TIPS yield is around 40 basis points in Model NL but are much smaller at around 7 basis points in Models L-I and L-II. The fitting errors are found to have substantial serial correlations. For example, in the case of the 5-year TIPS, we obtain a weekly AR(1) coefficient of 0.96, 0.75, and 0.75 for Models NL, L-I, and L-II, respectively. The finding of serial correlation in term structure fitting errors are however a fairly common phenomenon, and have been noted by Chen and Scott (1993) and others.

[Insert Table 3 about here.]

Overall Fit

Panel A of Table 4 reports some test statistics that compare the overall fit of the three models. We first report two information criteria commonly used for model selection, the Akaike Information Criterion (AIC) and the Bayesian Information Criterion (BIC). Both information criteria are minimized by the most general model, Model L-II.

We also report results from likelihood ratio (LR) tests of the two restricted models, Models NL and L-I, against their more general counterparts, Model L-I and L-II, respectively. Compared to Model L-I, Model NL imposes the restriction $\tilde{\gamma} = 0$. The standard likelihood ratio test does not apply here because the nuisance parameters, $\tilde{\kappa}$, $\tilde{\mu}$, $\tilde{\lambda}_0$ and $\tilde{\lambda}_1$, are not identified under the null (Model NL) and appear only under the alternative (Model L-I).[24] Here we follow Garcia and Perron (1996) and calculate a conservative upper bound for the significance of the likelihood ratio test statistic as suggested by Davies (1987), the details of which is outlined in Appendix C. Apply this procedure to testing the null of Model NL against the alternative of Model L-I gives an estimate of the maximal p value of essentially zero, hence Model NL is overwhelmingly rejected in favor of Model L-I. With the LR statistic estimated as $-2\log\left[L(\mathrm{NL})/L(\mathrm{L{-}I})\right] = 7911.18$ with 1 degree of freedom, we feel confident that using alternative econometric procedures to deal with the nuisance parameter problem is unlikely to overturn the rejection.

The LR test of Model L-I against Model L-II, on the other hand, is fairly standard. Based on the likelihood estimates of the two models, we obtain a LR statistic of

$$-2\log\left[L(\mathrm{L{-}I})/L(\mathrm{L{-}II})\right] = 20.79,$$

and are able to reject Model L-I in favor of Model L-II at the 1% level based on a χ^2_3 distribution.

[Insert Table 4 about here.]

5.2 Fitting TIPS Yields and TIPS BEI

The estimated standard deviations of TIPS measurement errors reported in Table 3 suggest that Model NL has trouble fitting the TIPS yields. A better understanding of the problem can be seen by comparing the top panels from Figures 5 and 6, which plot the actual and model-implied TIPS yields and TIPS BEI as well as the model-implied true real yields and true BEI for Models NL and L-II, respectively.[25] By construction, the model-implied TIPS yields (TIPS BEI) and the model-

[24] For discussions on testing with nuisance parameters, see, for example, Davies (1977, 1987, 2002) and Andrews and Ploberger (1994, 1995).

[25] Model-implied true breakevens are calculated as the difference between model-implied nominal yields and model-implied real yields of comparable maturities. Model-implied values are calculated using smoothed estimates of the state variables. Results for Model L-I are broadly similar to those for Model L-II and are reported in the internet

implied real yields (BEI) coincide under Model NL.

The top left panel of Figures 5 shows that Model NL attributes the decline in 10-year TIPS yields from 1999 to 2004 to a broad downward shift in real yields since the early 1990's, with real yields estimated to have come down from a level as high as 7% in the early 90's to about 2% around 2003. Over the same period, the 10-year nominal yield declined by less than 5% from around 9% to a little over 4%, which Model NL attributes almost entirely to a lower real yield, leaving little room for lower inflation expectation or lower inflation risk premiums. However, the literature generally finds that long-term inflation expectations likely have edged down during this period,[26] and it is hard to imagine economic mechanisms that would generate such a large decline in long-term real yields. Furthermore, although Model NL matches the general trend of TIPS yields during this period, it has problem fitting the time variations, frequently resulting in large fitting errors, especially in the early part of the sample and again during the recent financial crisis. In contrast, the top left panel of Figure 6 shows a less pronounced and more gradual decline in real yields based on Model L-II, and the model is able to fit TIPS yields almost perfectly, as shown by the juxtaposition of the red and black lines.

In addition, the top right panel of Figure 5 shows that the 10-year BEI implied by Model NL, which by construction should equal the 10-year TIPS BEI, appears too smooth compared to the actual data and misses most of its short-run variations. The poor fitting of the TIPS BEI highlights the difficulty that the 3-factor model has in fitting nominal and TIPS yields simultaneously. In contrast, the 10-year BEI implied by Models L-II, shown in the top right panel of Figure 6, exhibits substantial variations that closely match those of the actual TIPS BEI. In particular, the model-implied and the TIPS-based BEI plunge toward the end of 2008 following the Lehman collapse, consistent with reports of substantial liquidation of TIPS holdings over this period.[27]

To quantify the improvement in terms of the model fit, Panels B and C of Table 4 provide three goodness-of-fit statistics for TIPS yields at the 5-, 7- and 10-year maturities and TIPS BEI at the

appendix.

[26] See Kozicki and Tinsley (2006), for example.

[27] For a brief account of the episode, see Hu and Worah (2009).

7- and 10-year maturities, respectively. The first statistic, CORR, is the simple sample correlation between the fitted series and its data counterpart. The next two statistics are based on one-step-ahead model prediction errors from the Kalman Filter, $\mathrm{v_t}$, defined in Equation (65) in Appendix B, and are designed to capture how well each model can explain the data without resorting to large exogenous shocks or measurement errors. In particular, the second statistic is the root mean squared prediction errors (RMSE), and the third statistic is the coefficient of determination (R^2), defined as the percentage of in-sample variations of each data series explained by the model:

$$R^2 = 1 - \frac{\sum_{t=2}^{T} v_t^2}{\sum_{t=2}^{T} (y_t - \overline{y})^2}, \tag{50}$$

where y_t is the observed series and $\overline{y}$ denotes its sample mean.[28]

All three statistics suggest that allowing a TIPS liquidity premium component improves the model fit for raw TIPS yields and even more so for TIPS BEI. For example, the correlation between model-implied and actual 10-year TIPS breakevens rises from 18% to over 98% once we move from Model NL to the other models. Although Model NL generates a respectable sample correlation for TIPS yields of around 93%, the seemingly reasonable fit is only achieved by assuming large exogenous shocks to the state variables, as can be seen from the larger RMSEs and lower R^2s compared with the other two models. The fit of Model NL for TIPS BEI is even worse, with a R^2 of -13% at the 10-year maturity, compared to R^2s of more than 89% at both maturities.

[Insert Figures 5 and 6 about here.]

5.3 Matching Survey Inflation Forecasts

It is conceivable that a model with more parameters like Model L-II could generate smaller in-sample fitting errors for variables whose fit is explicitly optimized but produce undesirable implications for variables not used in the estimation. To check this possibility, we examine how closely

[28] Unlike in a regression setting, a negative value of R^2 could arise because the model expectation and the prediction errors are not guaranteed to be orthogonal in a small sample.

the model-implied inflation expectations mimic survey-based inflation forecasts, which are not used in our estimation. Ang, Bekaert, and Wei (2007) provide recent evidence that survey inflation forecasts outperforms various other measures of inflation expectations in predicting future inflation. In addition, survey inflation forecast has the benefit of being a real-time, model-free measure, and hence not subject to model estimation errors or look-ahead biases that could affect measures based on in-sample fitting of realized inflation.[29]

Panel D of Table 4 reports the three goodness-of-fit statistics, CORR, RMSE and R^2, for 1- and 10-year ahead inflation forecasts from the SPF. Neither Model NL nor Model L-I generates inflation expectations that agree well with survey inflation forecasts, as can be seen from the large RMSEs and small and even negative R^2 statistics. In contrast, Model L-II implies inflation expectations that are more than 70% correlated with their survey counterparts, generates smaller RMSEs at both horizons, and is able to explain a large amount of sample variations in survey inflation forecasts.

A visual comparison of the model-implied inflation expectations and survey forecasts help shed more light on those results. The middle panels of Figure 5, which plot Model NL-implied 1- and 10-year inflation expectation together with the survey forecasts, suggest that Model NL fails to capture the downward trend in survey inflation forecasts during much of the sample period, especially at the 10-year horizon, and implies a 10-year inflation expectations that moved little over the sample period. This is the flip side of the discussions in Section 5.2, where we see a Model-NL-implied 10-year real rate that is too variable and is used by the model to explain the entire decline in nominal yields during the 1990s. Overall, the near-constancy of the long-term inflation expectation is the most problematic feature of Model NL. In contrast, as can be seen from the middle panels of Figure 6, Model L-II produces 1- and 10-year inflation expectations that show a more visible downward trend, consistent with the survey evidence. In the internet appendix, we

[29] Alternatively, we could compare the out-of-sample forecasting performance of various models. However, we doubt the usefulness of such an exercise for two reasons. First and primarily, the sample available for carrying out such an exercise is extremely limited due to the relatively short sample of TIPS. In addition, the large idiosyncratic fluctuations associated with commonly used price indices would lead to substantial sampling variability in any metric of forecast performance we use and further complicate the inference problem.

show that Model L-I generates inflation expectations that match the downward trend of the survey forecasts but are much more volatile, which explains its poor fit to survey inflation forecasts as seen in Panel D of Table 4.

Finally, the bottom left panels of Figures 5 and 6 plot model-implied 1- and 10-year inflation risk premiums implied by Models NL and L-II, respectively. One immediately notable feature is that Model NL implies an inflation risk premium that is negative and increasing over time in the 1990-2013 period. In contrast, most existing studies find positive inflation risk premiums on average, as noted in Section 2. Furthermore, studies such as Clarida and Friedman (1984) indicate that the inflation risk premium likely was positive and substantial in the early 1980s and probably has come down since then. Indeed, the two models that allow for a liquidity premium, Models L-I and L-II, both generate 10-year inflation risk premiums that are mostly positive and fluctuate in the 0 to 0.5% range over the same sample period. The short-term inflation risk premiums implied by these two models, on the other hand, are fairly small, consistent with our intuition.

5.4 Summary

In summary, we find that Model NL, which equates TIPS yields with true underlying real yields, fares poorly along a number of dimensions, including generating a poor fit with TIPS yields and BEI, as well as inflation expectations and inflation risk premiums with counterintuitive properties. In contrast, models that allow for TIPS liquidity premiums show improvements along those dimensions. In particular, Model L-II produces short- and long-term inflation expectations that agree quite well with survey forecasts, suggesting it is important to allow for a systematic component in TIPS liquidity premiums. In the remainder of our analysis we'll be mainly focusing on this model.

6 TIPS Liquidity Premium

6.1 Model Estimates of TIPS Liquidity Premiums

The TIPS liquidity premiums implied by Model L-II are plotted in the bottom right panels of Figure 6 for maturities of 5, 7 and 10 years. Three things are worth noting from this graph: First, liquidity premiums exhibit substantial time variations at all maturities. The substantial variability at maturities as long as 10 years is in part due to the fact that the independent liquidity factor is estimated to be quite persistent under the risk-neutral measure. As can be seen from Table 3, the risk-neutral persistence of the liquidity factor, $\tilde{\kappa}^* = \tilde{\kappa} + \tilde{\sigma}\tilde{\lambda}_1$, is estimated to be very small at around 0.1 in all models and is tightly estimated, with a standard error of about 0.006. In contrast, the persistence parameter under the physical measure, $\tilde{\kappa}$, is not as precisely estimated, with its value and the associated standard error both estimated to be around 0.2.

Second, the term structure of implied TIPS liquidity premiums is downward sloping during the 2001-2004 and 2008-2011 periods. Technically, a market price of risk on the independent liquidity factor that is on average positive, as is the case here, would contribute to a downward-sloping term structure of TIPS liquidity premium.[30] This is in contrast to the standard one-factor interest rate models, where the market price of interest rate risk is typically found to be negative leading to an upward-sloping term structure of the term premium.

Finally, TIPS liquidity premiums were fairly high (1.5-2% range) when TIPS were first introduced but had been steadily declining until around 2004, likely reflecting the maturing process of a relatively new financial instrument. Liquidity premiums surged to record-high levels ($\sim$3.5%) after the Lehman bankruptcy, reflecting a sharp increase in transaction and funding costs of TIPS

[30] For example, it is straightforward to show that under Model L-I, the slope of the TIPS liquidity premium curve is given by

$$\frac{\partial L_{t,\tau}}{\partial \tau}|_{\tau=0} = 0.5\tilde{\kappa}^*(\tilde{\mu}^* - \tilde{x}_t),$$

the unconditional mean of which is given by

$$0.5\tilde{\kappa}^*(\tilde{\mu}^* - \tilde{\mu}) = -0.5\tilde{\sigma}(\tilde{\lambda}_0 + \tilde{\lambda}_1\tilde{\mu}).$$

where the equality comes from Equation (56). Therefore, if the average market price of liquidity risk, $\tilde{\lambda}_0 + \tilde{\lambda}_1\tilde{\mu}$, is positive, the term structure of the liquidity premium will be on average downward-sloping.

as well as heightened aversion towards liquidity risks.[31] The large flight-to-safety flows into the nominal Treasury market likely also contributed to the larger liquidity differential between nominal Treasuries and TIPS that led to a higher liquidity discount in TIPS relative to its nominal counterpart. Similarly sharp rise in liquidity premium and/or illiquidity measures were seen in other key markets during the recent financial crisis, including equity and corporate bond markets.[32] Outside those two periods, TIPS liquidity premiums appear low and stationary, fluctuating between -50 and 50 basis points.

6.2 Link to Observable Liquidity Measures

The behavior of the liquidity premiums seen in Figure 6 and described above seems broadly consistent with the perception that TIPS market liquidity conditions had improved over time till the inception of the recent financial crisis. However, given the unobserved nature of the liquidity factor in our model, one may question whether it is indeed capturing TIPS liquidity rather than other aspects of TIPS yields that are orthogonal to nominal Treasury yields. In this section, we verify the validity of our liquidity premium estimates by linking them to various observable measures of TIPS liquidity. One immediate difficulty we face is the lack of precise, real-time measures of liquidity conditions in the TIPS market. For example, as shown in the top left panel of Figure 7, one widely used measure of illiquidity, the bid-ask spread, only became available for TIPS in 2003 from TradeWeb. A TIPS liquidity measure that is available over a longer sample is the relative trading volumes of TIPS versus nominal coupon Treasury securities.[33] As can be seen from the top right panel of Figure 7, the trading volumes in TIPS remained low compared to nominal Treasury coupon securities up to mid 2004 and then rose substantially, suggesting steady improvement

[31] These estimates are somewhat larger than those in Pflueger and Viceira (2013) but broadly in line with those in Abrahams, Adrian, Crump, and Moench (2013).

[32] See, for example, Bao, Pan, and Wang (2011), Dick-Nielsen, Feldhütter, and Lando (2012), and Brennan, Huh, and Subrahmanyam (2012).

[33] We construct the measure as 13-week averages of weekly dealer transaction volumes in TIPS divided by those in nominal coupon Treasury securities using data reported by primary dealers and collected by the Federal Reserve Bank of New York under Government Securities Dealers Reports (FR-2004). This measure has been used in Pflueger and Viceira (2013) and Abrahams, Adrian, Crump, and Moench (2013).

in TIPS liquidity during the pre-crisis sample period.[34] The rise coincides roughly with the decline in our TIPS liquidity premium estimates over the same period, whose correlations with the TIPS relative trading volumes are about -80% over the period of January 1999 to December 2007.

[Insert Figure 7 about here]

Another observable measure of TIPS liquidity used in the literature is the average absolute fitting errors from the Svensson TIPS yield curve, plotted in the middle left panel of Figure 7. The logic behind this measure derives from funding constraints and limits to arbitrage that prevent investors from eliminating the deviations of yields from their fundamental values as measured from the fitted yield curve. Similar measures have been used by Fontaine and Garcia (2012) and Hu, Pan, and Wang (2012) to measure the liquidity of nominal Treasury securities, and by Grishchenko and Huang (2013) for TIPS. It is plausible that during a financial crisis, capital becomes more scarce and risk aversion run higher, leaving significant arbitrage opportunities unexploited. According to this measure, liquidity conditions in the TIPS market were fine until the inception of the financial crisis, when they suddenly deteriorated.

All three measures mentioned so far capture current liquidity conditions in the TIPS market but not expected future liquidity conditions or the risk premiums investors place on bearing such liquidity risks. They also capture the absolute level of (il)liquidity of TIPS rather than the relative liquidity against that of nominal Treasury securities. To capture those additional components, we also examine two closely-related measures based on asset prices. The first such measure is the difference between TIPS and off-the-run nominal asset swap (ASW) spreads obtained from Barclays.[35] ASW spreads vary over time and across securities according to the perceived default and liquidity risk of the underlying securities. Because both nominal Treasury and TIPS are usually considered free of default risks, their ASW spreads can be regarded as a good market-based measure of the liquidity premiums in those assets. Consistent with their relative liquidity, we usually

[34] The increase in TIPS trading volumes during the 2003-2004 period may be partially driven by the increased market attention to inflation risk amid a booming economy and rising oil prices.

[35] In a fixed-income asset swap, one party exchanges the fixed-rate cash flows from the underlying security for a floating-rate cash flow, where the floating rate is typically quoted as 6-month LIBOR plus a spread—the asset swap spread.

observe that ASW spreads became increasingly more negative as we move across asset classes from TIPS to off-the-run nominal Treasuries and then to on-the-run nominal Treasuries. The difference between TIPS and nominal Treasury ASW spreads would therefore be an ideal measure of the relative illiquidity of TIPS.[36] Unfortunately TIPS ASWs only started trading in 2006; we therefore use the difference between the off-the-run and the on-the-run 10-year nominal Treasury ASW spreads from JP Morgan as an approximation when studying the full-sample. The correlation between the two measures of ASW spread differences is 0.93 since 2006 when both are available. As can be seen from the middle right and the bottom left panels of Figure 7, both measures spiked during the crisis, reflecting general funding pressures leading to a dramatic widening between prices of securities with only small liquidity differentials.

The second forward-looking measure of TIPS liquidity premiums is the difference between inflation swaps and TIPS BEI, available since late 2004 and plotted in the bottom right panel of of Figure 7.[37] As noted by Campbell, Shiller, and Viceira (2009), in theory this measure is linked to the difference between the TIPS and nominal ASW spreads through a no-arbitrage relationship; empirically, the two measures have a high correlation of 88% in the post-2006 sample when both are available, although the swap-BEI difference exhibits a smaller spike during the recent crisis and also an earlier return to its normal levels afterwards.

Panel A of Table 5 reports the pairwise correlations between Model L-II-implied TIPS liquidity premium estimates and the six observable liquidity measures mentioned above. As expected, model liquidity premium estimates are negatively correlated with the relative transaction volumes and positively correlated with all other liquidity measures. The magnitude of the correlation is the largest for the ASW spread-based measures and the swap-BEI difference, suggesting a large part of the variations in model liquidity premium are related to future liquidity risks and liquidity

[36] Such a measure is used in a recent study by Campbell, Shiller, and Viceira (2009), which focuses on a more recent sample of July 2007 to April 2009.

[37] Inflation swaps are over-the-counter swap contracts under which one party pays a fixed rate—the inflation swap rate—and the other party pays a floating rate that equals the realized inflation rate. Inflation swap rates are usually above the TIPS BEI at the same maturity due to liquidity differences between the TIPS and nominal Treasury markets as well as the lack of liquidity in the inflation swaps market. Fleckenstein, Longstaff, and Lustig (Forthcoming) and Christensen and Gillan (2011) examine these explanations in more details.

risk premiums. Among the six observable liquidity measures, the three forward-looking measures are highly correlated with each other and with the TIPS bid-ask spread and the TIPS curve fitting errors. The relative TIPS trading volumes are not highly correlated with the other measures and show counterintuitive positive correlations with the bid-ask spread, although those correlations are higher and all with the expected signs during the pre-crisis period, suggesting that trading activities may have become a less important determinant of TIPS liquidity once the TIPS market has matured.

[Insert Table 5 about here]

To quantify the effects of these factors on our estimates of TIPS liquidity premiums, we run univariate and multivariate regressions of the 10-year TIPS liquidity premiums from Model L-II on various liquidity measures. Panel B of Table 5 examines the full sample using the three measures that are available over the entire sample period—the relative TIPS-nominal trading volumes, the nominal on- and off-the-run ASW spread difference, and the average TIPS curve fitting errors. The coefficients on the three variables all carry the expected signs and are statistically significant. In general, TIPS liquidity premiums are lower when TIPS transaction volumes rise relative to those of nominal Treasury securities, when nominal on- and off-the-run ASW spreads trade closer to each other, and/or when TIPS yields show smaller deviations from their fundamental values. Together these three variables explain 77% of the variations in 10-year TIPS liquidity premium estimates.

Panel C of Table 5 expands the regressions to include all six liquidity measures over the sample period since September 20, 2006, when all measures became available. Again, most coefficients are of the expected sign and statistically significant, except for the TIPS bid-ask spread and the swaps-BEI difference, which are significant on their own but become insignificant when all TIPS liquidity measures are included. The magnitude of the coefficients on the relative TIPS trading volumes is smaller than those from the full-sample regressions, consistent with the correlation pattern documented above and with the intuition that the "growing pains" of TIPS market is a more important story in the earlier part of the sample. In the univariate regression, the coefficient on the TIPS fitting errors nearly doubled and the regression now explains a much larger portion of liquidity premium variations, reflecting the important role of funding constraints and limits to arbitrage in driving TIPS liquidity during the recent crisis. The same three variables as in Panel B explain a slightly higher percentage (79.1%) of the variations in model liquidity premium estimates, while this percentage rises further to 86.3% when all observable liquidity measures (except the differences between nominal on- and off-the run ASW spreads) are included.

The results from Table 5 confirm that the model-implied liquidity premiums are indeed capturing current and expected future relative liquidity conditions in the TIPS market as well as the associated liquidity risk premiums. We caution, however, that this type of regressions should be viewed only as a rough gauge of the relationship between the observed measures and the liquidity premiums embedded in TIPS yields, as quantities like bid-ask spreads and trading volumes cannot be expected to have a simple linear relationship with the liquidity premium. As a result, we feel that the latent-liquidity-factor approach used in this paper has the advantage of being more flexible than rigidly linking the liquidity premiums to one or more observable measures. Nonetheless, our analysis confirms that the difference between the TIPS and nominal ASW spreads stands out as the most promising real-time, observable measure of TIPS liquidity premiums, at least over the short sample period when it is available.

6.3 Economic Significance of TIPS Liquidity Premiums

In this section we assess the economic significance of the TIPS liquidity premiums by examining the proportions of variations in TIPS yields and TIPS BEI that can be attributed to variations in TIPS liquidity premiums. Using Equations (2) and (30), we can decompose TIPS yields, $y_{t,\tau}^{\mathcal{T}}$, and TIPS BEI, $BEI_{t,\tau}^{\mathcal{T}}$, into different components:

$$y_{t,\tau}^{\mathcal{T}} = y_{t,\tau}^{R} + L_{t,\tau}, \qquad BEI_{t,\tau}^{\mathcal{T}} = I_{t,\tau} + \wp_{t,\tau} - L_{t,\tau}, \tag{51}$$

where $y_{t,\tau}^{R}$ is the true underlying real yield, $L_{t,\tau}$ is the TIPS liquidity premium, $I_{t,\tau}$ is expected inflation over the next τ periods, and $\wp_{t,\tau}$ is the inflation risk premium. Table 6 reports the in-sample variance decomposition results based on Equation (51) and Model L-II estimates. A time series plot of the decomposition is shown in Figure 8.

[Insert Table 6 about here.]

[Insert Figure 8 about here.]

For TIPS yields, real yields dominate TIPS liquidity premiums in accounting for the time variations. In comparison, TIPS liquidity premiums are more important in driving TIPS BEI variations, explaining 10-15% of its variations across the three maturities, although expected inflation still accounts for the majority of time variations in TIPS BEI. Our results suggest that one should be especially cautious in interpreting variations in TIPS BEI solely in terms of changes in inflation expectation or inflation risk premiums.

7 Application to the U.K.

To asses the robustness of our findings, in this section we apply our models to data from the U.K., one of the first developed countries to issue indexed securities when it introduced the index-linked gilts in 1981.[38] Those gilts are indexed to the retail price index (RPI) and have an indexation lag of about 8 months. The inflation-indexed gilt market grew rapidly and now makes up about one quarter of the inflation-adjusted total amount of government bonds outstanding, far higher than the ratio in the U.S. One distinguishing feature of the U.K. index-linked gilt market is the dominance of institutional investors—defined as pension funds and combined insurance companies.[39]

Given the longer history of the index-linked gilt market and the much higher demand for such securities from market participants, we expect that liquidity premiums would account for a smaller portion of the variations in index-linked gilt yields than those in TIPS yields over our sample period. Indeed, as can be seen from Panel A of Table 1, three conventional gilt yields explain a

[38] For a brief history of the U.K. index-linked gilt market, see Deacon, Derry, and Mirfendereski (2004).

[39] According to quarterly MQ5 release on "Investment by Insurance Companies, Pension Funds and Trusts" from the U.K. Office of National Statistics, institutional investors hold about 70% of all outstanding index-linked gilts, with pension fund holdings accounting for more than 40% of the market, throughout the entire sample period.

much higher proportion of variations in the level of the 10-year BEI than in the U.S., although that proportion has declined notably since the onset of the recent crisis. In addition, as shown in Figure 9, two observable measures of the liquidity in the index-linked gilt market—the difference between the 10-year U.K. inflation swap rate (from Bloomberg) and the 10-year BEI, and the difference between the ASW spreads on index-linked gilts and those on the nominal counterparts (from Barclays)—have generally fluctuated at lower levels than those from the U.S. over the period since 2004 when they became available; both measures show notable increases during the financial crisis, similar to what we observe for the U.S.

We estimate the models over a monthly sample of January 1993 to March 2013. The starting date was motivated by the desire to avoid a potential structural break when the U.K. first adopted an inflation target in October 1992 following its departure from the Exchange Rate Mechanism.[40] We use end-of-month zero-coupon yields on conventional gilts with maturities of 3 and 6 months and 1, 2, 4, 7, and 10 years and those on index-linked gilts with maturities of 5, 7, and 10 years, as implied by the spline curves proposed by Anderson and Sleath (2001) and maintained by the Bank of England (BoE).[41] The inflation measure is the quarterly seasonally-adjusted RPI. Figure 10 plots both sets of yields, the BEI rates, and realized inflation. In addition to yields and inflation, we use survey forecasts of the 3-month interbank rate 3-month and 1-year ahead, available monthly from the Consensus Forecast survey. Unfortunately no long-range survey forecast of the short rate is available. To better pin down model parameters, we also use survey forecasts of inflation over the next year, available monthly, and over the next 5 to 10 years, available twice a year in April and October from the same survey.[42]

We estimate all three models with either three or four nominal factors; the potentially larger number of nominal factors is motivated by the observation that the first three principal components

[40] The BoE targeted a range of 1 to 4% from October 1992 to June 1995, when it adopted a point target of 2.5%. The targeted price is the retail price index excluding mortgage interest payments (RPIX) between October 1992 and December 2003 and the Consumer Prices Index (CPI) inflation since. CPI inflation in the U.K. typically runs about 1 percentage point below RPIX inflation.

[41] Data downloaded from http://www.bankofengland.co.uk/statistics/Pages/yieldcurve/archive.aspx.

[42] Inflation forecasts were for RPI inflation prior to 1997 and for RPIX inflation since. The two measures track each other relatively closely.

explain a slightly smaller proportion of nominal yields in the UK than in the U.S., as can be seen from Panel B of Table 1.[43] The best model is Model L-I with four nominal factors and the results from this model are shown in Figure 11. As shown in the bottom right panel of that figure, liquidity premiums in indexed gilt yields fluctuates within plus and minus 50 basis points prior to the crisis, confirming our earlier conjecture that, on average, index-linked gilt yields contain smaller liquidity premiums than TIPS over that period. Liquidity premiums jumped to about 250 basis points at the height of the crisis, a level only slightly lower than those of TIPS, reflecting common funding liquidity shortages across those two countries with well-integrated financial markets. Much of the decline in 10-year BEI over the sample period is attributed to lower inflation risk premiums, while inflation expectations remained stable. Overall these results offer encouraging evidence that our model works well in a setting different from what it is originally designed for. Nonetheless, given the large differences in the institutional setup of the two markets, a more careful examination of the UK data than what is done here is needed to verify the findings in this section.

8 Conclusion

We document in this paper the existence of a TIPS-specific factor that appears important for explaining TIPS yield and TIPS BEI variations, and provide evidence that this factor likely reflects an (il)liquidity premium in TIPS yields.

We develop a joint no-arbitrage term structure model of nominal and TIPS yields incorporating two different specifications of the TIPS liquidity premiums. We show that ignoring the liquidity premium components leads to a poor model fit of TIPS yields, TIPS BEI, and survey inflation forecasts. Our estimated TIPS liquidity premiums were fairly large ($\sim 1\%$) until about 2003 and fluctuated within narrow ranges between then and the onset of the crisis, consistent with the common perception that TIPS market liquidity was poor when TIPS were first introduced but it had steadily improved over time. TIPS liquidity premium estimates shot up to near 350 basis points

[43] Joyce, Lildholdt, and Sorensen (2010) also assume that four factors drive the U.K. nominal term structure, although only two factors drive indexed gilt yields in their model.

after the Lehman bankruptcy, reflecting the stringent funding liquidity conditions over that period. The model liquidity premium estimates are shown to be linked to changes in some observable measures of TIPS liquidity. When applied to the U.K. data, our model uncovers liquidity premiums on index-linked gilts that are fairly low in normal times, consistent with the larger size of the index-linked gilt market, but spiked to about 250 basis points at the height of the financial crisis.

TIPS BEI has increasingly gained attention as a measure of investors' inflation expectations that is available in real-time and at high frequencies. However, our results raise caution in interpreting movement in TIPS BEI solely in terms of changing inflation expectations, as substantial liquidity premiums and inflation risk premiums could drive a large wedge between the two, as demonstrated vividly during the recent financial crisis. A better understanding of the determinants of TIPS liquidity premiums and the sources of its variation remains an interesting topic for future research.

A TIPS Liquidity Premium

Since $\tilde{x}_t$ is independent of the other state variables in x_t, the first term on the right-hand side of Equation (31) can be written as the sum of two components:

$$
\begin{aligned}
& -(1/\tau)\log E_t^Q\left(\exp\left(-\int_t^{t+\tau}(r_s^R+l_s)ds\right)\right) \\
= & -(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}\tilde{\gamma}\tilde{x}_s ds}) - (1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}(\rho_0^R+(\rho_1^R+\gamma)'x_s)ds}) \qquad (52)
\end{aligned}
$$

The first component can be solved to be

$$
-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}\tilde{\gamma}\tilde{x}_s ds}) = \tilde{a}_\tau + \tilde{b}_\tau \tilde{x}_t \qquad (53)
$$

where $\tilde{a}$ and $\tilde{b}$ has the familiar form of factor loadings in a one-factor Vasicek model:

$$
\begin{aligned}
\tilde{a}_\tau &= \tilde{\gamma}\left[(\tilde{\mu}^* - \frac{\tilde{\sigma}^2}{2\tilde{\kappa}^*})(1-\tilde{b}_\tau) + \frac{\tilde{\sigma}^2}{4\tilde{\kappa}^*}\tau\tilde{b}_\tau^2\right] && (54) \\
\tilde{b}_\tau &= \tilde{\gamma}\frac{1-\exp(-\tilde{\kappa}^*\tau)}{\tilde{\kappa}^*\tau}, && (55)
\end{aligned}
$$

in which the risk-neutral $\tilde{\kappa}^*, \tilde{\mu}^*$ are given by

$$
\tilde{\kappa}^* = \tilde{\kappa} + \tilde{\sigma}\tilde{\lambda}_1, \qquad \tilde{\mu}^* = (\tilde{\kappa}\tilde{\mu} - \tilde{\sigma}\tilde{\lambda}_0)/\tilde{\kappa}^*, \qquad (56)
$$

The second component can be shown to take the form

$$
-(1/\tau)\log E_t^Q(e^{-\int_t^{t+\tau}(\rho_0^R+(\rho_1^R+\gamma)'x_s)ds}) = a_\tau^{\mathcal{T}} + b_\tau^{\mathcal{T}\prime} x_t \qquad (57)
$$

where $a^{\mathcal{T}}$, $b^{\mathcal{T}}$ are given by

$$
\begin{aligned}
a_\tau^{\mathcal{T}} &= -A_\tau^{\mathcal{T}}/\tau, && (58) \\
b_\tau^{\mathcal{T}} &= -B_\tau^{\mathcal{T}}/\tau, && (59)
\end{aligned}
$$

where

$$
\begin{aligned}
\frac{dA_\tau^{\mathcal{T}}}{d\tau} &= -\rho_0^R + B_\tau^{\mathcal{T}\prime}\left(\mathcal{K}\mu - \Sigma\lambda_0^R\right) + \frac{1}{2}B_\tau^{\mathcal{T}\prime}\Sigma\Sigma' B_\tau^{\mathcal{T}} && (60)\\
\frac{dB_\tau^{\mathcal{T}}}{d\tau} &= -\left(\rho_1^R + \gamma\right) - \left(\mathcal{K} + \Sigma\Lambda^R\right)' B_\tau^{\mathcal{T}} && (61)
\end{aligned}
$$

with initial conditions $A_0^{\mathcal{T}} = 0$ and $B_0^{\mathcal{T}} = 0_{n\times 1}$. Taken together, we have that

$$
\begin{aligned}
L_{t,\tau}^s &= \left(\tilde{a}_\tau + a_\tau^{\mathcal{T}}\right) + \begin{bmatrix} b_\tau^{\mathcal{T}\prime} & \tilde{b}_\tau \end{bmatrix} \begin{bmatrix} x_t \\ \tilde{x}_t \end{bmatrix} - y_t^R \\
&= \left[\tilde{a}_\tau + (a_\tau^{\mathcal{T}} - a_\tau^R)\right] + \begin{bmatrix} ,\cdot\ (b_\tau^{\mathcal{T}} - b_\tau^R)' & \tilde{b}_\tau \end{bmatrix} \begin{bmatrix} x_t \\ \tilde{x}_t \end{bmatrix} && (62)
\end{aligned}
$$

where the second equality comes from Equation (24).

B Kalman Filter and Likelihood Function

We use the Kalman filter to compute optimal estimates of the unobservable state factors based on all available information. For example, given the initial guess for the state factors $\widehat{\mathrm{x}}_0$, it follows from the state equation (44) that the optimal estimate of the state factor x_t at time $t = h$ is given by

$$
\widehat{\mathrm{x}}_{h,0} \triangleq E(\mathrm{x_h} \mid \Im_0) = \mathrm{G_h} + \Gamma_\mathrm{h}\widehat{\mathrm{x}}_0,
$$

which implies that we carry the error of the initial guess to all subsequent estimations. More generally, we have

$$
\widehat{\mathrm{x}}_{t,t-h} \triangleq E(\mathrm{x_t} \mid \Im_\mathrm{t-h}) = \mathrm{G_h} + \Gamma_\mathrm{h}\widehat{\mathrm{x}}_\mathrm{t-h,t-h}. \qquad (63)
$$

for any time period t. The variance-covariance matrix of the estimation error can be derived as

$$\begin{aligned} Q_{t,t-h} &\triangleq E\left[(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})'\right] \\ &= E\left\{\left[\Gamma_h\left(\mathrm{x_{t-h}} - \widehat{\mathrm{x}}_{\mathrm{t-h}}\right) + \eta_t^{\mathrm{x}}\right]\left[\Gamma_h\left(\mathrm{x_{t-h}} - \widehat{\mathrm{x}}_{\mathrm{t-h}}\right) + \eta_t^{\mathrm{x}}\right]'\right\} \\ &= \Gamma_h Q_{t-h,t-h}\Gamma_h' + \Omega_{t-h}^{\mathrm{x}}, \end{aligned} \tag{64}$$

where $\Omega_{t-h}^{\mathrm{x}} = E\left[\eta_t^{\mathrm{x}}\eta_t^{\mathrm{x}\prime}\right]$. Given any forecast for $\mathrm{x_t}$, we can compute a forecast for the observable variables based on all time $t-h$ information:

$$\widehat{\mathrm{y}}_{t,t-h} \triangleq E\left[\mathrm{y_t} \mid \Im_{\mathrm{t-h}}\right] = A + B\widehat{\mathrm{x}}_{t,t-h},$$

the forecast error of which is given by

$$\mathrm{v_t} \triangleq \mathrm{y_t} - \widehat{\mathrm{y}}_{t,t-h} = B(\mathrm{x_t} - \widehat{\mathrm{x}}_{t,t-h}) + \mathrm{e_t}. \tag{65}$$

The conditional variance-covariance matrix of this estimation error can then be computed as

$$\begin{aligned} V_{t,t-h} &\triangleq E\left\{(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}})(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}})'\right\} \\ &= BQ_{t,t-h}B' + \Omega_t^e \end{aligned} \tag{66}$$

where

$$\Omega_t^e = E\left[\mathrm{e_t}\mathrm{e_t'}\right].$$

The next step is to update the equation for the state variables. Before doing this, we need to recover the distribution of $\mathrm{x_t} \mid \mathrm{y_t}$. The conditional covariance between the forecasting errors for state variables and that for observation variables takes the form of

$$\begin{aligned} V_{t,t-h}^{xy} &\triangleq E\left\{(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}})(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})'\right\} \\ &= BE\left[(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})(\mathrm{x_t} - \widehat{\mathrm{x}}_{\mathrm{t,t-h}})'\right] \\ &= BQ_{t,t-h} \end{aligned} \tag{67}$$

The conditional joint distribution for $\mathrm{y_t}$ and $\mathrm{x_t}$ is therefore

$$\begin{bmatrix} \mathrm{y_t} \\ \mathrm{x_t} \end{bmatrix} \mid \Im_{t-h} \sim N\left(\begin{bmatrix} a + F\widehat{\mathrm{x}}_{t,t-h} \\ \widehat{\mathrm{x}}_{t,t-h} \end{bmatrix}, \begin{bmatrix} BQ_{t,t-h}B' + \Omega^e_t & BQ_{t,t-h} \\ Q_{t,t-h}B' & Q_{t,t-h} \end{bmatrix} \right),$$

which implies that conditional on $\mathrm{y_t}$, $\mathrm{x_t}$ is also distributed normal:

$$\mathrm{x_t} \mid \mathrm{y_t} \sim N\left(\widehat{\mathrm{x}}_{t,t}\,, Q_{t,t}\right),$$

where

$$\widehat{\mathrm{x}}_{t,t} = \widehat{\mathrm{x}}_{t,t-h} + Q_{t,t-h}B'V^{-1}_{t,t-h}\left(\mathrm{y_t} - \widehat{\mathrm{y}}_{\mathrm{t,t-h}}\right) \tag{68}$$

$$Q_{t,t} = Q_{t,t-h} - Q_{t,t-h}B'V^{-1}_{t,t-h}BQ_{t,t-h}; \tag{69}$$

The variance-covariance matrix of the updated state vector, $Q_{t,t}$, will be smaller than the previous estimate, $Q_{t,t-h}$, due to the new information coming in through the observation of $\mathrm{y_t}$. We estimate the parameters by maximizing the log-likelihood function

$$\begin{aligned} \sum_{t=h}^{Th} \log f\left(\mathrm{y_t}|\Im_{\mathrm{t-h}}\right) = &-\frac{T}{2}\log(2\pi) - \frac{1}{2}\sum_{t=h}^{Th}\log|V_{t,t-h}| \\ &- \frac{1}{2}\sum_{t=h}^{Th}\left[\left(\mathrm{y_t} - \mathrm{A} - \mathrm{B}\widehat{\mathrm{x}}_{\mathrm{t,t-h}}\right)' V^{-1}_{t,t-h}\left(\mathrm{y_t} - \mathrm{A} - \mathrm{B}\widehat{\mathrm{x}}_{\mathrm{t,t-h}}\right)\right]. \end{aligned} \tag{70}$$

C Davies (1987) Likelihood Ration Test Statistic

This section describes the details in constructing the Davies (1987) Likelihood Ration Test Statistic mentioned in Section 5.1. Denote by θ the vector of nuisance parameters of size s, and define the likelihood ratio statistic as a function of θ:

$$LR\left(\theta\right) = 2\left[\log L_1\left(\theta\right) - \log L_0\right],$$

where $L_1(\theta)$ is the likelihood value of the alternative model for any admissible values of the nuisance parameters $\theta \in \Omega$, and L_0 is the maximized likelihood value of the null model. For an estimated LR value of M, Davies (1987) derives an upper bound for its significance as

$$\Pr\left[\sup_{\theta \in \Omega} LR(\theta) > M\right] < \Pr\left[LR(\theta) > M\right] + VM^{\frac{1}{2}(s-1)} \exp^{-(M/2)} \frac{2^{-s/2}}{\Gamma(s/2)}$$

where $\Gamma(.)$ represents the Gamma function and V is defined as

$$V = \int_{\Omega} \left| \frac{\partial LR(\theta)}{\partial \theta} \right| d\theta.$$

Garcia and Perron (1996) further assumes that the likelihood ratio statistic has a single peak at $\widehat{\theta}$, which reduces V to $2M^{\frac{1}{2}}$.

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Table 1: Factors Driving Nominal and Inflation-Indexed Bonds

Panel A: Regressing 10-year Breakevens on Nominal Yields: Adjusted R^2 (in%)

U.S.			U.K.		
Sample	in level	in weekly changes	Sample	in level	in monthly changes
Full Sample	6.0	39.3	Full Sample	73.2	39.0
1999-2007	30.2	57.3	1993-2007	88.5	58.0
2008-2013	5.4	26.7	2008-2013	32.4	36.3

Panel B: Variances Explained by Principal Components (Full sample, in %)

	U.S.		U.K.	
PC	nominal yields only	nominal and TIPS yields	nominal yields only	nominal and indexed yields
1st	70.6	65.2	66.0	54.2
2nd	92.7	86.1	82.7	77.8
3rd	97.6	94.1	95.8	88.0
4th	99.2	97.6	99.3	96.5

Panel C: Correlation of Principal Components (Full sample)

		U.S. nominal and TIPS yields				U.K. nominal and indexed yields			
		PC1	PC2	PC3	PC4	PC1	PC2	PC3	PC4
nominal	PC1	**0.95**	-0.28	-0.15	0.01	**0.94**	-0.34	-0.04	0.04
yields	PC2	0.18	**0.86**	-0.48	0.02	0.14	**0.52**	-0.76	0.35
alone	PC3	0.03	0.07	0.17	**0.98**	0.07	0.22	**0.54**	**0.81**
	PC4	0.01	0.02	0.06	-0.05	0.07	0.20	0.12	-0.18

The left columns in **Panel A** report adjusted R^2's from regressions of 10-year TIPS breakeven inflation rates on 3-month, 2-year, and 10-year nominal Treasury yields over the full weekly sample of 1/6/99 to 3/27/13 and over two sub-samples of 1/6/99-12/26/07 and 1/2/08-3/27/13. The right columns report similar results for the U.K., where 10-year index-linked gilt-based breakeven inflation rates are regressed on 6-month, 2-year, and 10-year conventional gilt yields over the full monthly sample of 1/31/93-3/31/13 and over two sub-samples of 1/31/93-12/31/07 and 1/31/08-3/31/13. **Panel B** reports cumulative percentages of variances of weekly or monthly changes in nominal yields only or nominal and indexed bond yields combined that are explained by their first four principal components for both the U.S. (left columns) and the U.K. (right columns). The corresponding in-sample pairwise correlations between the two sets of principal components for each country are reported in **Panel C**.

Table 2: Summary of Models

Model	Restrictions and Identifications
Model NL	$\gamma = 0_{3\times 1}, \quad \tilde{\gamma} = 0, \quad \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$
Model L-I	$\gamma = 0_{3\times 1}, \quad \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unrestricted
Model L-II	$\gamma, \tilde{\gamma}, \tilde{\kappa}, \tilde{\mu}, \tilde{\lambda}_0, \tilde{\lambda}_1$ unrestricted

This table lists the parameter restrictions placed on the three models we estimate, including a model assuming zero TIPS liquidity premiums (Model NL), a model assuming TIPS liquidity premiums that are orthogonal to the other stare variables in the economy (Model L-I), and a model allows correlation between TIPS liquidity premiums and the other state variables.

Table 3: Selected Parameter Estimates

	Model NL		Model L-I		Model L-II	
State Variables Dynamics						
$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t$						
$\mathcal{K}_{11}$	0.8604	(0.9710)	0.8227	(0.3514)	0.8106	(0.8240)
$\mathcal{K}_{22}$	0.1316	(0.0534)	0.1318	(0.0536)	0.1311	(0.0549)
$\mathcal{K}_{33}$	1.4528	(1.1180)	1.4652	(0.3786)	1.4833	(0.9599)
$100 \times \Sigma_{21}$	-0.7358	(1.0533)	-0.7685	(0.5582)	-0.7724	(1.0446)
$100 \times \Sigma_{31}$	-4.4381	(16.7353)	-4.0872	(4.2439)	-3.8970	(11.1829)
$100 \times \Sigma_{32}$	-0.9482	(0.2934)	-0.9848	(0.2389)	-0.9827	(0.2644)
Log Price Level						
$d\log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp},\ \pi(x_t) = \rho_0^{\pi} + \rho_1^{\pi\prime} x_t$						
ρ_0^{π}	0.0269	(0.0016)	0.0355	(0.0031)	0.0297	(0.0031)
$\rho_{1,1}^{\pi}$	-0.0209	(1.7085)	1.0897	(0.5285)	0.4419	(1.2812)
$\rho_{1,2}^{\pi}$	0.1072	(0.0607)	0.3698	(0.0806)	0.1952	(0.1091)
$\rho_{1,3}^{\pi}$	-0.2214	(0.4512)	0.0064	(0.1497)	0.0569	(0.3339)
$100 \times \sigma_{q,1}$	-0.0712	(0.0451)	-0.1033	(0.0411)	-0.0923	(0.0559)
$100 \times \sigma_{q,2}$	-0.0038	(0.0709)	0.0627	(0.0613)	0.0933	(0.0759)
$100 \times \sigma_{q,3}$	-0.0601	(0.0672)	0.0167	(0.0610)	-0.0100	(0.0688)
$100 \times \sigma_q^{\perp}$	0.9136	(0.0280)	0.9209	(0.0295)	0.8916	(0.0285)
TIPS Liquidity Premium						
$l_t = \tilde{\gamma}\tilde{x}_t + \gamma' x_t,\ d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma} dW_t,\ \tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1 \tilde{x}_t.$						
$\tilde{\gamma}$			0.9500	(0.0287)	0.9650	(0.0292)
$\tilde{\kappa}$			0.2199	(0.2227)	0.2211	(0.2294)
$\tilde{\mu}$			0.0119	(0.0102)	0.0042	(0.0111)
$\tilde{\lambda}_0$			0.1973	(0.4285)	0.1278	(0.2954)
$\tilde{\sigma}\tilde{\lambda}_1$			-0.0876	(0.2228)	-0.0867	(0.2296)
γ_1					-0.3621	(1.1991)
γ_2					-0.1987	(0.1226)
γ_3					0.1671	(0.4911)
Measurement Errors: TIPS Yields						
$100 \times \delta_{\mathcal{T},5y}$	0.6006	(0.1104)	0.0900	(0.0042)	0.0892	(0.0042)
$100 \times \delta_{\mathcal{T},7y}$	0.4707	(0.1263)	-0.0000	(1905.3160)	0.0000	(113.6434)
$100 \times \delta_{\mathcal{T},10y}$	0.4112	(0.0742)	0.0683	(0.0037)	-0.0680	(0.0037)
Measurement Errors: Survey Forecasts of Nominal Short Rate						
$100 \times \delta_{f,6m}$	0.1900	(0.0147)	0.1901	(0.0149)	0.1902	(0.0148)
$100 \times \delta_{f,12m}$	0.2979	(0.0224)	0.2980	(0.0228)	0.2981	(0.0227)

This table reports selected parameter estimates and standard errors for all three models we estimate. Standard errors are calculated using the BHHH formula and are reported in parentheses. Complete parameter estimates can be found in the internet appendix.

Table 4: Specification Tests

		Model NL	Model L-I	Model L-II
Panel A: Overall model fit				
No. of parameters		42	47	50
Log likelihood		72,768.06	76,723.65	76,734.05
AIC		-145,452.12	-153,353.30	-153,368.10
BIC		-145,237.89	-153,113.56	-153,113.06
LR p-value		0.00*	0.04	
Panel B: Fitting TIPS yields				
5-year	CORR (in %)	93.21	99.87	99.87
	RMSE	0.60	0.16	0.16
	R^2 (in %)	84.79	98.86	98.86
7-year	CORR (in %)	94.57	100.00	100.00
	RMSE	0.48	0.13	0.13
	R^2 (in %)	88.63	99.16	99.16
10-year	CORR (in %)	95.34	99.86	99.86
	RMSE	0.43	0.14	0.14
	R^2 (in %)	88.32	98.75	98.75
Panel C: Fitting TIPS Breakeven Inflation				
7-year	CORR (in %)	36.08	100.00	100.00
	RMSE	0.47	0.11	0.11
	R^2 (in %)	10.62	95.12	95.08
10-year	CORR (in %)	17.74	98.53	98.53
	RMSE	0.40	0.12	0.12
	R^2 (in %)	-12.96	89.19	89.21
Panel D: Matching survey inflation forecasts				
1-year	CORR (in %)	44.05	64.00	77.06
	RMSE	0.65	0.80	0.43
	R^2 (in %)	3.91	-45.14	57.71
10-year	CORR (in %)	63.02	67.03	71.15
	RMSE	0.49	0.65	0.40
	R^2 (in %)	16.56	-48.02	43.62

This table reports various diagnostic statistics for the three models estimated. **Panel A** reports the number of parameters, the log likelihood, the Akaike information criterion (AIC), the Bayesian information criterion (BIC) values, and the p-value from a Likelihood Ratio test of the current model against the more general Model to its right, where the p-values reported for Models NL is the Davies (1987) upper bound. **Panels B to D** report three goodness-of-fit statistics for the 5-, 7- and 10-year TIPS yields, 7- and 10-year TIPS breakeven inflation and 1- and 10-year survey inflation forecasts, respectively, including the correlation between the fitted series and the data counterpart (CORR), the root mean squared prediction errors (RMSE), and the coefficient of determination (R^2) as defined in Equation (50).

Table 5: What Drives the TIPS Liquidity Premiums

Panel A: Pairwise Correlations

	Liquidity Premiums 5-year	Liquidity Premiums 7-year	Liquidity Premiums 10-year	Rel TIPS Trading Vol	TIPS Curve Fit Err	Nom On/Off ASW Diff	TIPS-Nom ASW Diff	Swaps-BEI Diff
Ten-year TIPS bid-ask spread	0.51	0.49	0.47	0.30	0.66	0.57	0.58	0.41
Relative TIPS transaction volume	-0.57	-0.57	-0.56		0.39	-0.28	-0.12	0.01
Average TIPS curve fitting err	0.41	0.40	0.38			0.60	0.83	0.69
Nominal On/off ASW spread diff	0.80	0.79	0.78				0.93	0.67
TIPS-nominal ASW spread diff	0.91	0.92	0.91					0.88
Inf swaps-BEI difference	0.74	0.76	0.79					

Panel B: Regression Analysis: Full Sample

	(1)	(2)	(3)	(4)
Constant	1.2286	0.1008	0.4598	0.8646
	(0.0371)	(0.0179)	(0.0196)	(0.0372)
Relative TIPS transaction volume	-0.4043			-0.4119
	(0.0218)			(0.0185)
Nominal On/off ASW spread diff		0.0311		0.0163
		(0.0009)		(0.0012)
Averge TIPS curve fitting error			0.0396	0.0360
			(0.0036)	(0.0032)
No. of observations	743	743	743	743
Adjusted R-squared	31.7%	60.7%	14.2%	77.2%

Panel C: Regression Analysis: Since 2006

	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Constant	0.0694	1.0066	0.0717	0.0148	-0.2715	-0.3300	0.6599	0.2576
	(0.0409)	(0.1249)	(0.0232)	(0.0214)	(0.0205)	(0.0357)	(0.0682)	(0.0632)
Ten-year TIPS bid-ask spread	0.1564							-0.0036
	(0.0140)							(0.0078)
Relative TIPS transaction volume		-0.2753					-0.3184	-0.2276
		(0.0591)					(0.0316)	(0.0279)
Average TIPS curve fitting err			0.0682				0.0466	0.0237
			(0.0030)				(0.0041)	(0.0037)
Nominal On/off ASW spread diff				0.0278			0.0122	
				(0.0010)			(0.0016)	
TIPS-nominal ASW spread diff					0.0187			0.0124
					(0.0005)			(0.0013)
Inf swaps-BEI difference						0.0260		0.0023
						(0.0011)		(0.0015)
No. of observations	341	341	341	341	341	341	341	341
Adjusted R-squared	27.0%	5.7%	61.1%	69.3%	83.3%	63.3%	79.1%	86.3%

Panel A reports the in-sample pairwise correlations between 5-, 7- and 10-year TIPS liquidity premium estimates implied by Model L-II and various TIPS liquidity measures, including the relative TIPS transaction volumes over nominal coupon securities, the difference between 10-year on- and off-the-run nominal Treasury ASW spreads, 10-year TIPS bid-ask spreads, the average difference between TIPS and nominal Treasury ASW spreads, the difference between the 10-year inflation swap rate and TIPS BEI, and the average TIPS curve fitting errors. Most data is weekly from Jan. 6, 1999 to Mar. 27, 2013, with the exceptions of TIPS bid-asp spreads (since Feb. 5, 2003), TIPS-nominal ASW spread differences (since Sep. 20, 2006), and inflation-TIPS BEI difference (since Aug 16, 2004). **Panel B** reports results from univariate and multivariate regressions of the model-implied 10-year TIPS liquidity premiums on the relative TIPS trading volume, the nominal on/off-the-run ASW spread difference, and the average TIPS curve fitting error using weekly data from Jan. 6, 1999 to Mar. 27, 2013. **Panel C** reports results from univariate and multivariate regression of the model-implied 10-year TIPS liquidity premium on all TIPS liquidity measures from Sep. 20, 2006 to Mar. 27, 2013.

Table 6: In-Sample Variance decomposition of TIPS Yields and TIPS BEI

	TIPS yield		TIPS BEI		
Maturity	real yield	liq prem	inf exp	inf risk prem	liq prem
5-year	0.9985	0.0015	0.7020	0.1392	0.1588
	(0.2609)	(0.2609)	(0.5122)	(0.2212)	(0.6377)
7-year	1.0021	-0.0021	0.7040	0.1704	0.1256
	(0.2533)	(0.2533)	(0.5267)	(0.2588)	(0.6767)
10-year	1.0183	-0.0183	0.6763	0.2103	0.1135
	(0.2444)	(0.2444)	(0.5088)	(0.2929)	(0.6842)

Note: This table reports the in-sample variance decompositions of TIPS yields into real yields and TIPS liquidity premiums, and of nominal yields into expected inflation, the inflation risk premiums and the negative of TIPS liquidity premiums, all based on Model L-II estimates. The variance decompositions of TIPS yields are calculated according to

$$1 = \frac{cov\left(y^{\mathcal{T}}_{t,\tau}, y^{R}_{t,\tau}\right)}{var\left(y^{\mathcal{T}}_{t,\tau}\right)} + \frac{cov\left(y^{\mathcal{T}}_{t,\tau}, L_{t,\tau}\right)}{var\left(y^{\mathcal{T}}_{t,\tau}\right)},$$

while the variance decompositions of the TIPS BEI are calculated according to

$$1 = \frac{cov\left(BEI^{\mathcal{T}}_{t,\tau}, I_{t,\tau}\right)}{var\left(BEI^{\mathcal{T}}_{t,\tau}\right)} + \frac{cov\left(BEI^{\mathcal{T}}_{t,\tau}, \wp^{I}_{t,\tau}\right)}{var\left(BEI^{\mathcal{T}}_{t,\tau}\right)} + \frac{cov\left(BEI^{\mathcal{T}}_{t,\tau}, -L_{t,\tau}\right)}{var\left(BEI^{\mathcal{T}}_{t,\tau}\right)}.$$

Standard errors calculated using the delta method are reported in parentheses.

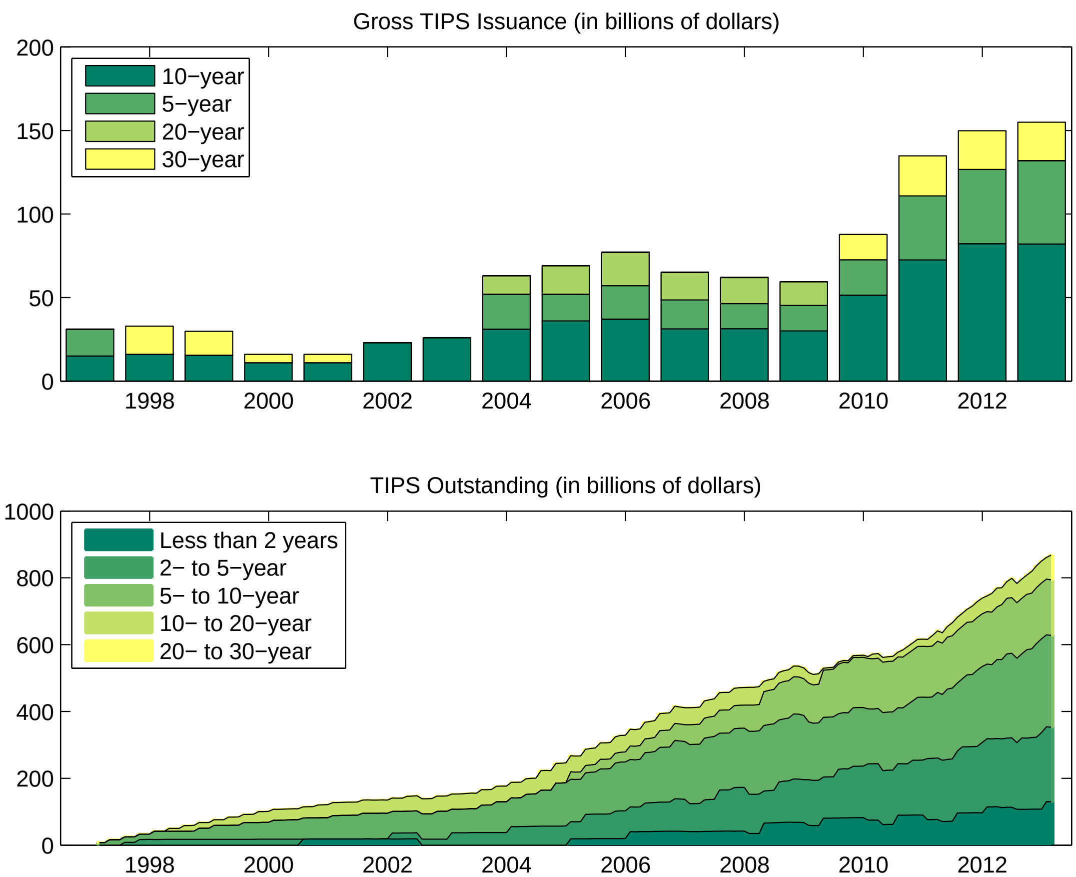


Figure 1: TIPS Issuance and Outstanding

The top panel plots gross TIPS issuance broken down by initial maturities of 10, 5, 20 and 30 years. The bottom panel plots TIPS outstanding broken down by remaining maturities, based on data reported in the Treasury's Monthly Statement of the Public Debt (MSPD).

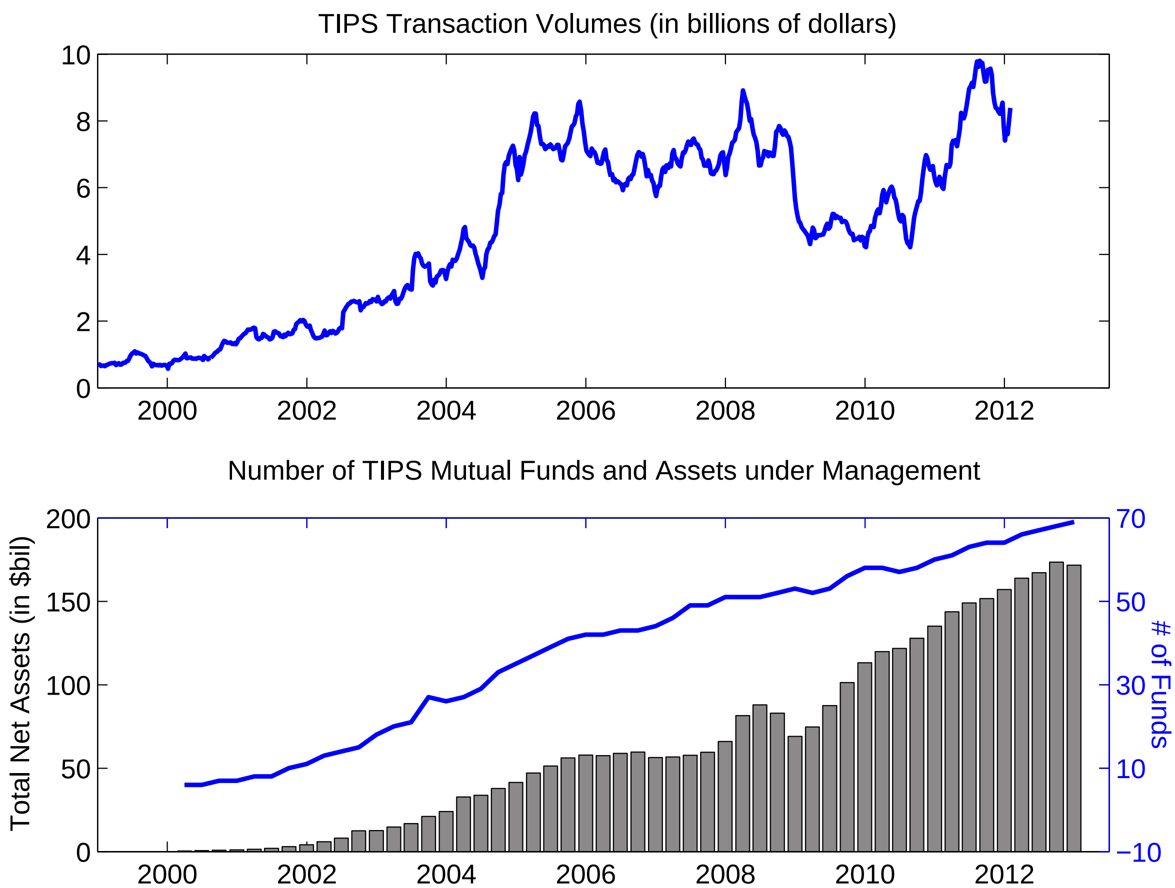


Figure 2: TIPS Transaction Volumes and TIPS Mutual Funds

The top panel plots the weekly TIPS transaction volumes, defined as 13-week moving average of weekly averages of daily TIPS transaction volumes reported by primary dealers in Government Securities Dealers Reports (FR-2004). The bottom panels plots number of TIPS mutual funds (right axis) and the total net assets under management (left axis).)

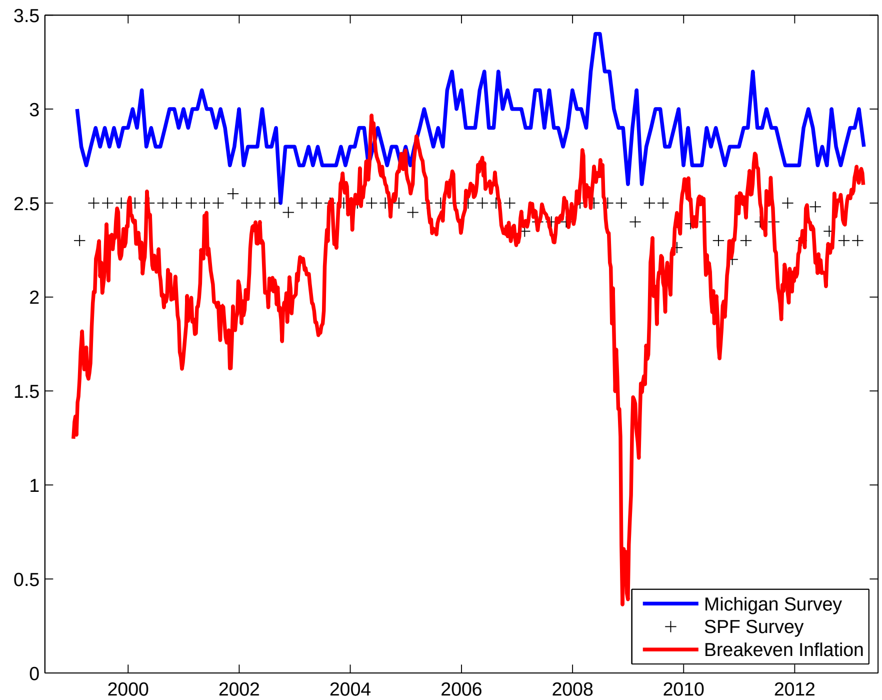


Figure 3: Survey Inflation Forecasts and TIPS Breakeven Inflation

This chart shows the 10-year TIPS breakeven inflation (red line), long-horizon Michigan inflation forecast (blue line), and 10-year SPF inflation forecast (black pluses).

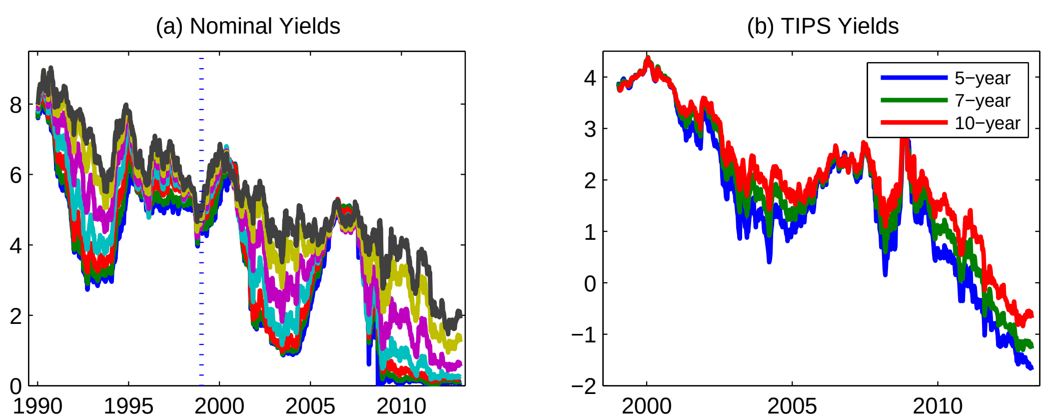


Figure 4: Nominal and TIPS Yields

Panel (a) plots the 3- and 6-month, 1-, 2-, 4-, 7- and 10-year nominal yields. Panel (b) plots the 5-, 7- and 10-year TIPS yields.

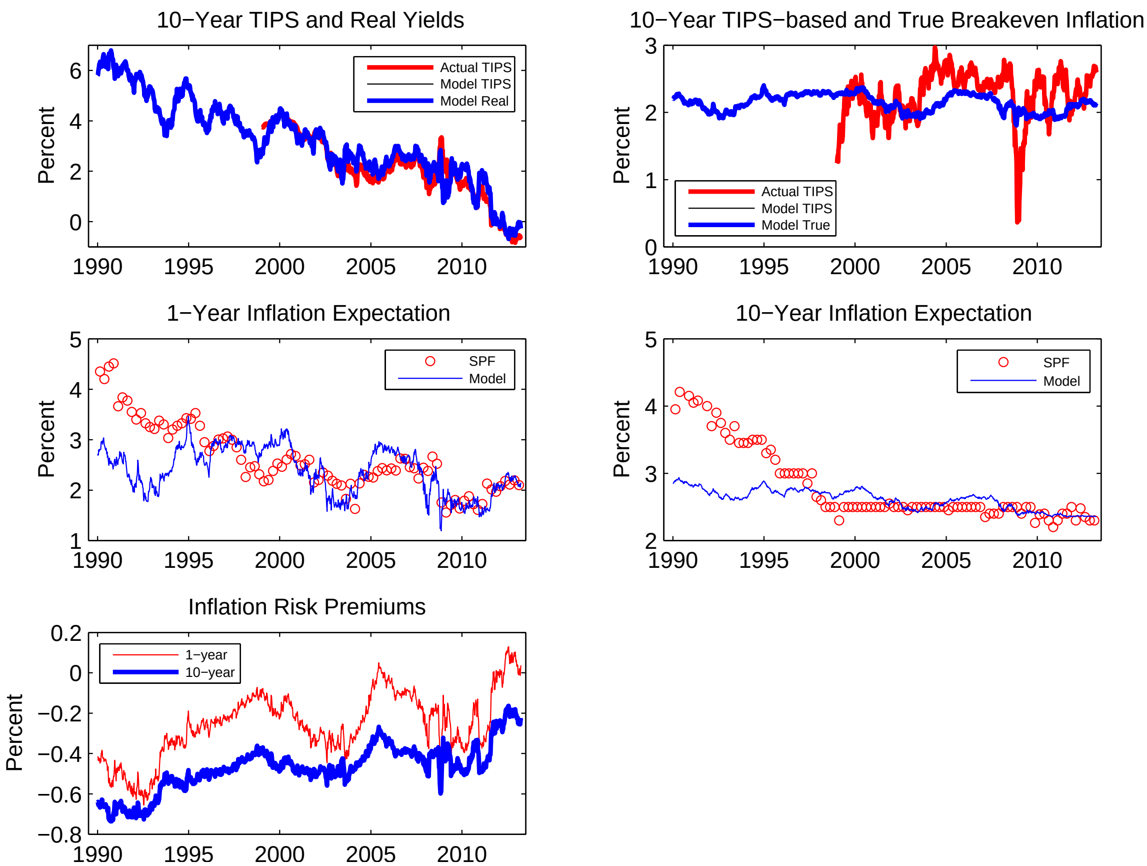


Figure 5: Model NL: Main Results

The top left panel plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The top right panel plots the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue). The middle panels plot the 1- and 10-year model-implied inflation expectation, respectively, together with their survey counterparts from the SPF. The bottom left panel plot the 1- and 10-year model-implied inflation risk premiums.

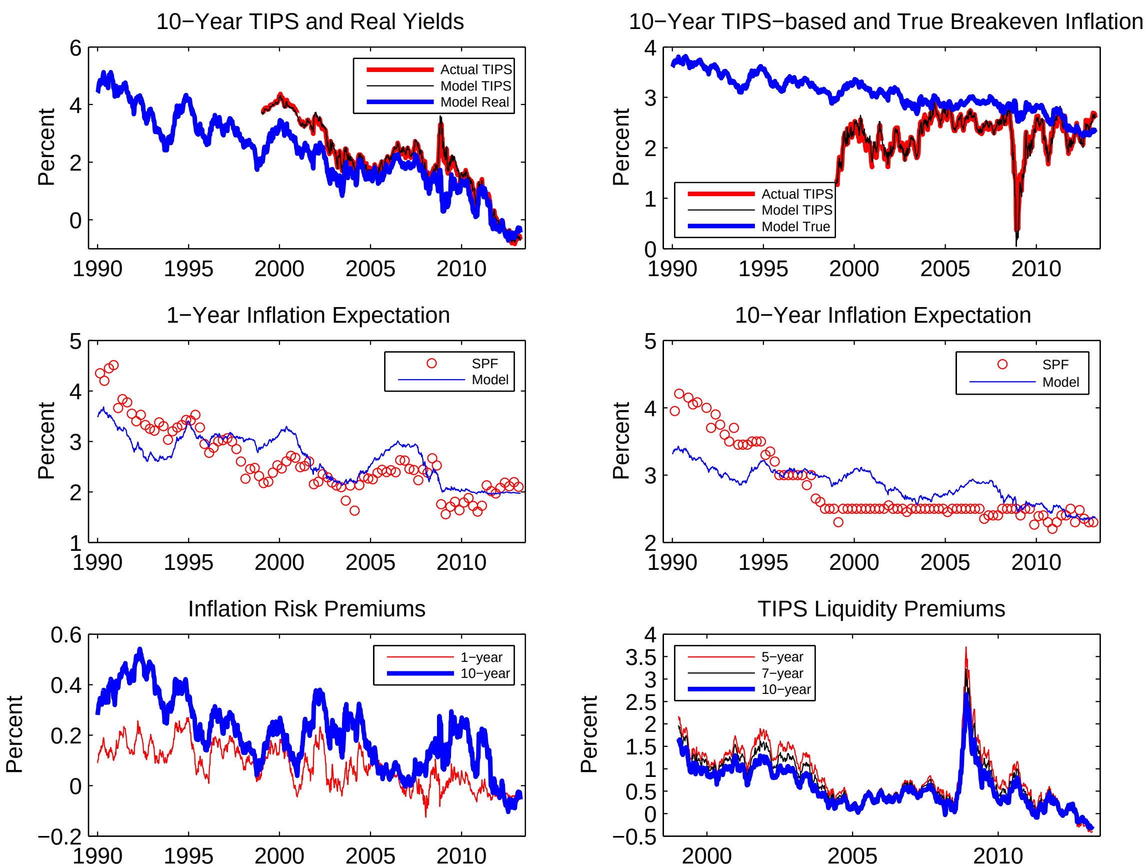


Figure 6: Model L-II: Main Results

The top left panel plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The top right panel plots the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue). The middle panels plot the 1- and 10-year model-implied inflation expectation, respectively, together with their survey counterparts from the SPF. The bottom left panel plot the 1- and 10-year model-implied inflation risk premiums. The bottom right panel plot the 5-, 7-, and 10-year model-implied TIPS liquidity premiums.

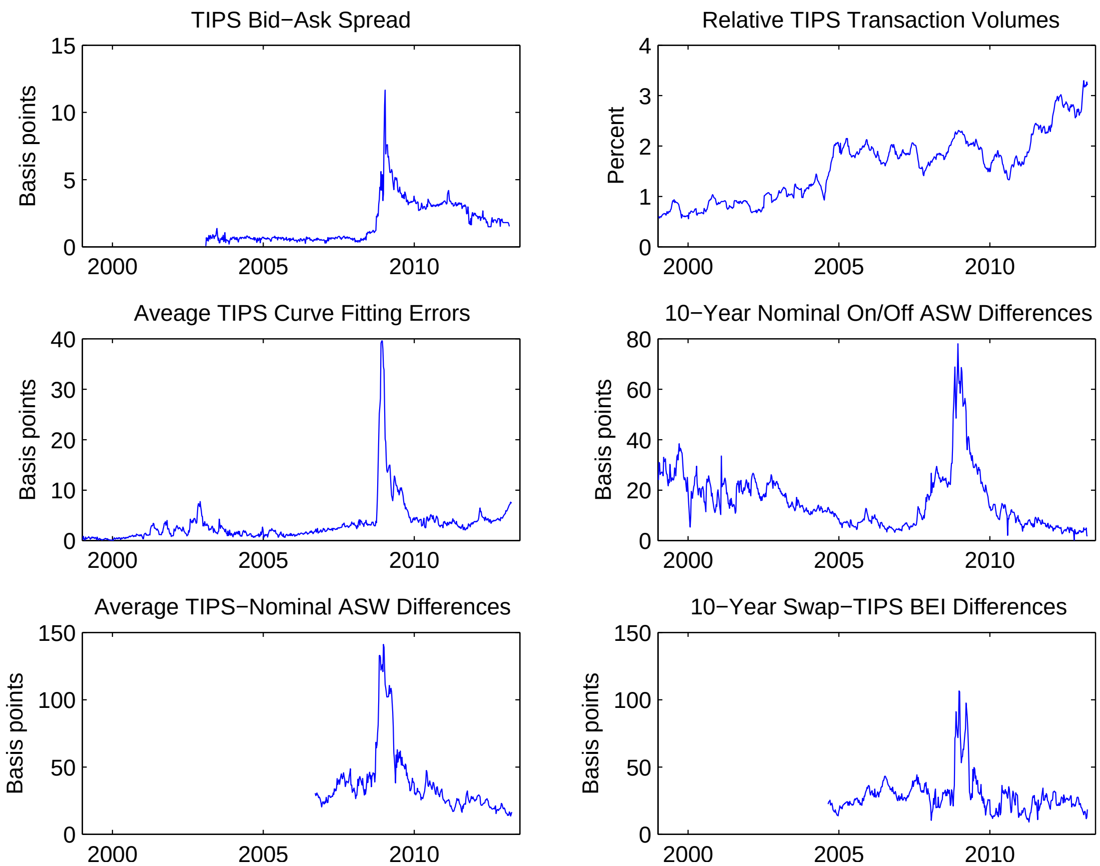


Figure 7: Observable Measures of TIPS Liquidity

This chart plots various measures of liquidity conditions in the TIPS market, including the 10-year TIPS bid-ask spread (top left), the relative TIPS trading volumes relative to those in nominal Treasury coupon securities (top right), the average mean fitting errors from the Svensson TIPS yield curve (middle left), the difference between the off-the-run and the on-the-run 10-year nominal Treasury par asset swap spreads (middle right), the average difference between TIPS and nominal Treasury asset swap spreads (bottom left), and the difference between 10-year inflation swap rate and the 10-year BEI (bottom right).

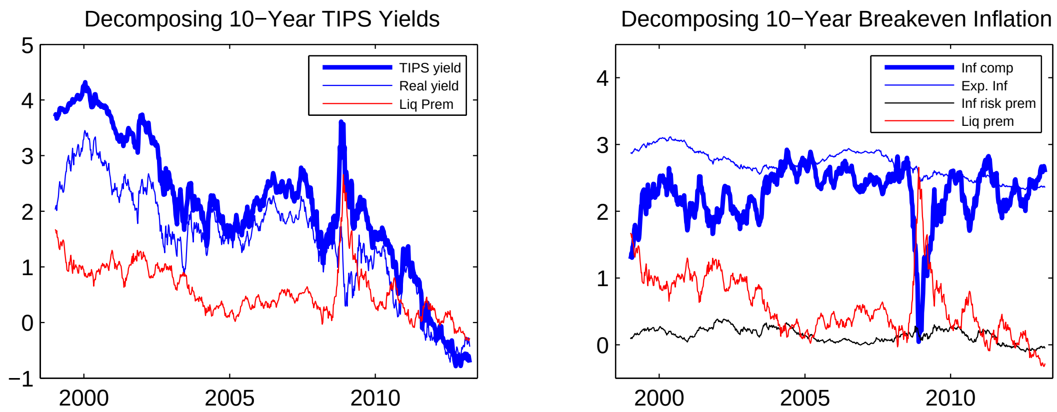


Figure 8: Decomposing TIPS Yields and TIPS Breakeven Inflation

The left panel decomposes the 10-year TIPS yields into the real yield and the TIPS liquidity premiums, while the right panel decomposes the 10-year TIPS breakeven inflation into the expected inflation, the inflation risk premium and the TIPS liquidity premium, all according to Equation (51).

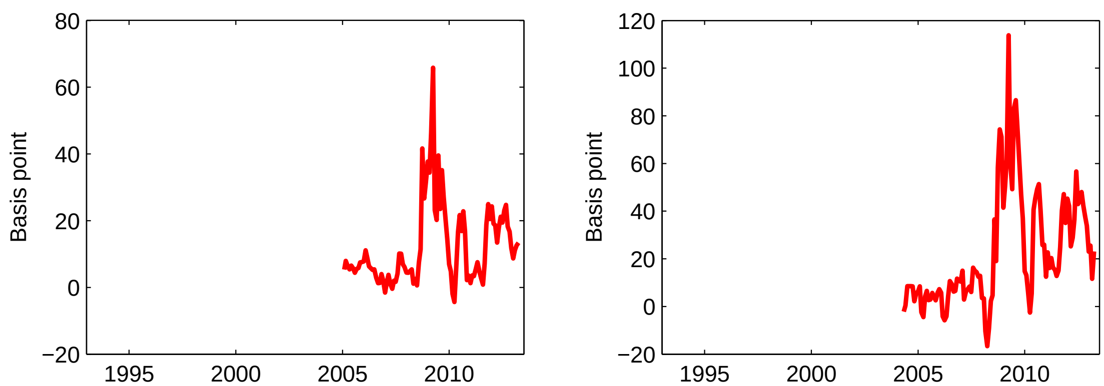


Figure 9: Observable Measures of Indexed Gilt Liquidity

This chart plots two observable measures of liquidity conditions in the index-linked gilt market, including the difference between index-linked and conventional gilt asset swap spreads (left panel), and the difference between the 10-year inflation swap rate and the 10-year BEI (right panel).

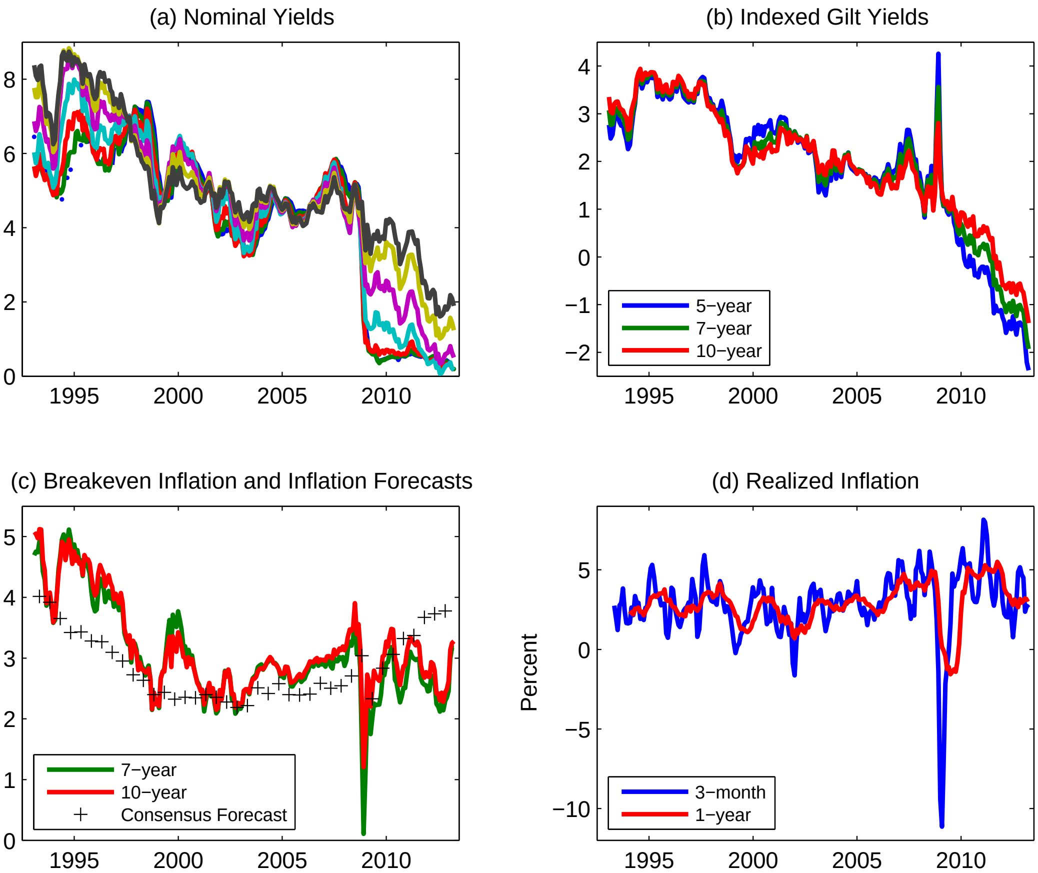


Figure 10: U.K. Yields, Breakeven Inflation, and Realized Inflation

Panel (a) plots the 3- and 6-month, 1-, 2-, 4-, 7- and 10-year nominal yields. Panel (b) plots the 5-, 7- and 10-year index-linked gilt yields. Panel (c) plots the 7- and 10-year breakeven inflation and the 10-year inflation forecast from Consensus Forecast. Panel (d) plots the 3-month and 1-year realized inflation based on the retail price index (RPI).

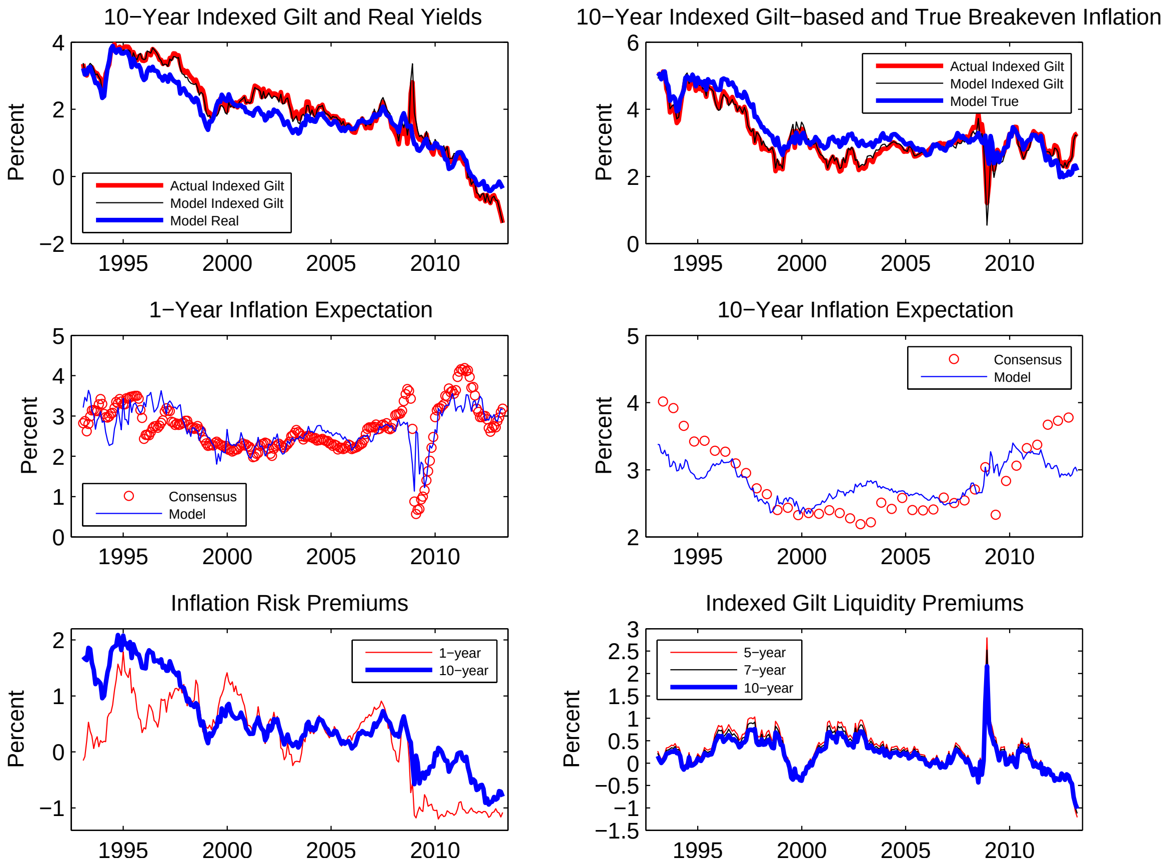


Figure 11: Model L-I Estimated with UK Data: Main Results

The red and black lines in the top left panel plot the actual and the fitted yields on 10-year index-linked gilts, respectively, while the blue line plots the 10-year model-implied real yields. The red and black lines in the top right panel plots the actual and the fitted breakeven inflation rates based on 10-year index-linked gilts, while the blue line plots the model-implied 10-year true breakevens. The middle panels plot the model-implied 1- and 10-year inflation expectation, respectively, together with the survey counterparts from Consensus Forecast. The bottom left panel plot the model-implied 1- and 10-year inflation risk premiums. The bottom right panel plot the model-implied liquidity premiums on 5-, 7-, and 10-year index-linked gilts.

Internet Appendix to
"Tips from TIPS: the Informational Content of Treasury Inflation-Protected Security Prices"

A. More on the TIPS Data

This appendix is devoted to a more detailed discussion of the TIPS data. Figure IA.1 shows the smoothed TIPS par yield curves on June 9, 2005 in the top panel and on June 9, 1999 in the bottom panel, together with the actual traded TIPS yields that were used to create the smoothed TIPS par-yield and zero-coupon curves. The smoothing is done by assuming that the zero-coupon TIPS yield curve follows the 4-parameter Nelson and Siegel (1987) functional form up to the end of 2003 and the 6-parameter Svensson (1995) functional form thereafter,[1] and minimizing the fitting error for the actual traded TIPS securities. The substantial increase in the number of points in the top panel reflects the growth of the TIPS market. Note that in 1999 there is essentially one data point on the curve between the 0 and 5 years maturity (corresponding to the 5-year TIPS issued in 1997), thus the TIPS term structure in the short-maturity region of 0-5 years cannot be determined reliably. With more points across the maturity spectrum in 2005, shorter maturity TIPS yields can be determined more reliably than in 1999.

Still, the analysis of the short-maturity TIPS are complicated by the indexation lag and seasonality issues. Note that the TIPS payments are indexed to the CPI index from 2.5 months earlier, thus TIPS yields contain some amount of realized inflation, often referred to as the "carry effect". The yield that is more relevant to investors is the one that takes out this realized inflation – the so-called carry-adjusted yields, which can be heuristically represented as

$$y_{t,\tau}^{\mathcal{T},CA} = y_{t,\tau}^{\mathcal{T}} + (\delta/\tau)\pi_{t-\delta,t}, \tag{IA.1}$$

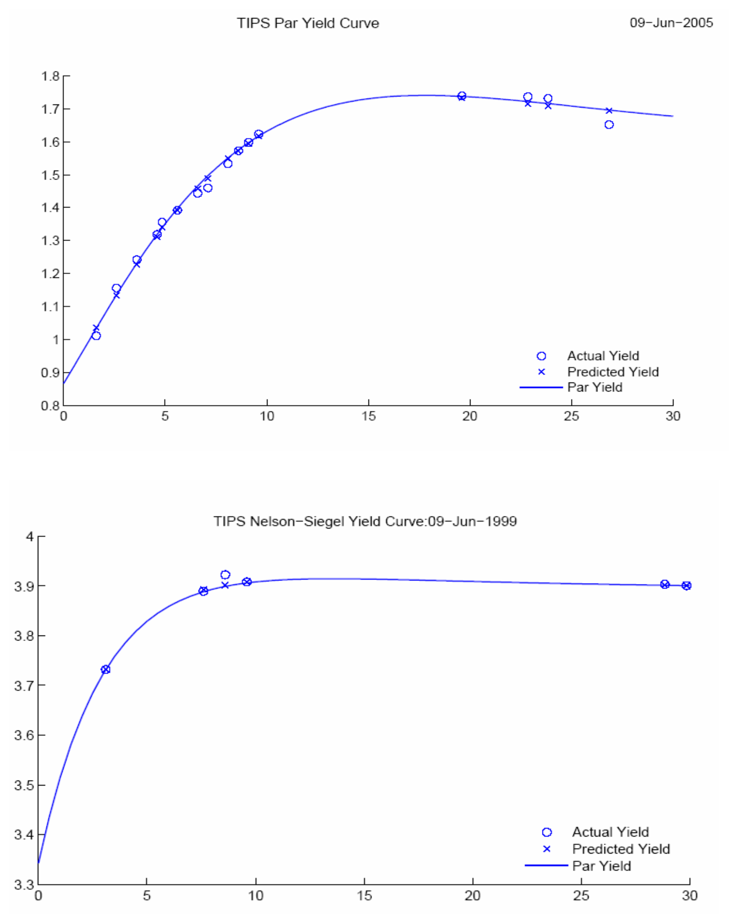
where $\pi_{t-\delta,t} = \log(Q_t/Q_{t-\delta})/\delta$ denotes the inflation realized between time $t-\delta$ and t, with $\delta = 2.5$ months.[2] Because the realized inflation $\pi_{t-\delta,t}$ can be quite volatile, the carry-unadjusted yield $y^{\mathcal{T}}$ and the carry-adjusted yield $y^{\mathcal{T},CA}$ can differ significantly, though the difference becomes smaller with increasing maturity, due to the δ/τ factor in Equation (IA.1). Figure IA.2 shows the carry-adjusted and unadjusted TIPS yields for 5-year and 10-year maturities. It can be seen that indeed the 5-year carry-adjusted and unadjusted TIPS yields show greater discrepancies than the 10-year ones. This has been particularly the case in 2005, during which large fluctuations in oil prices caused sharp short-term fluctuations in inflation. The expression (IA.1) for the carry adjustment is only a schematic one. The actual carry-adjustment is further complicated by the fact that TIPS is indexed to the seasonally-unadjusted CPI, rather than the seasonally-adjusted CPI. While one could in principle explicitly model seasonality (and carry effects) within the dynamic model of inflation and term structure, such a procedure may be more prone to specification errors than the case in which these effects are corrected at the input stage.[3]

As noted in the main text, since data reliability and indexation lags pose larger problems to shorter-maturity TIPS, in this paper we focus on long-maturity TIPS yield for which the effects of indexation lag and seasonality are less important. While the analysis of shorter-maturity TIPS yields is an important

[1]In comparison, the zero-coupon nominal yield curve is assumed to follow the 6-parameter Svensson (1995) functional form during the entire sample period. In the case of TIPS, however, there were not enough securities in the early years to pin down as many parameters. See Gürkaynak, Sack, and Wright (2007, 2010) for details.

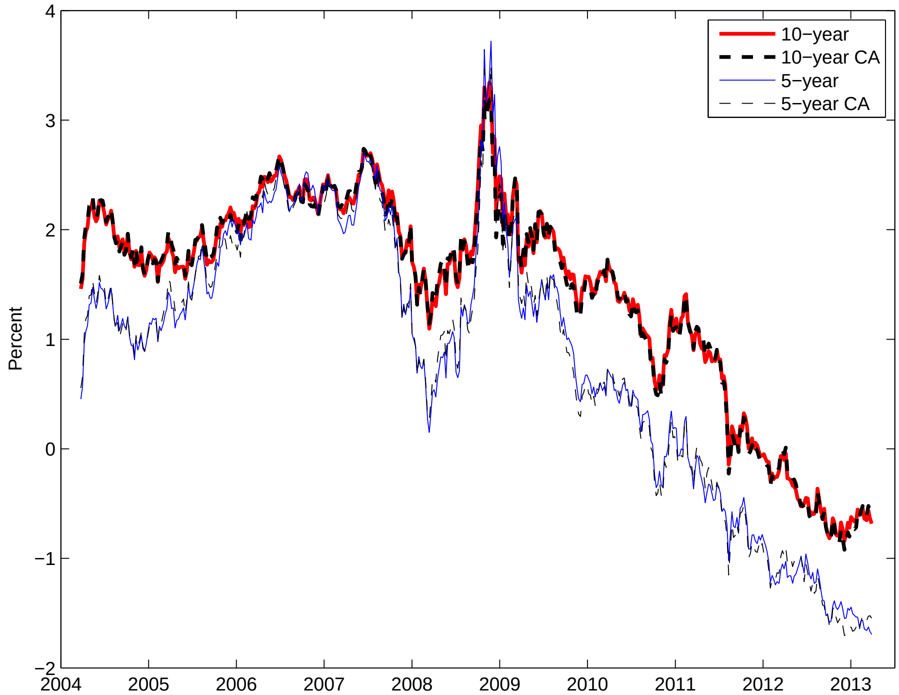
[2]Note that equation (IA.1) takes out realized inflation in the previous 2.5 months but makes no adjustment for the lack of inflation protection during the last 2.5 months prior to the maturity date.

[3]See Ghysels (1993) for a recent discussion of the Sims (1974)-Sargent (1978) debate that bears on this issue.



Note: The top (bottom) panel plots the fitted TIPS par yield curve together with individual TIPS yields on June 9, 2005 (June 9, 1999).

Figure IA.1. TIPS Yield Curves



Note: This figure plots 10-year carry-unadjusted (carry-adjusted) TIPS yields in red solid (black dashed) line and 5-year carry-unadjusted (carry-adjusted) TIPS yields in blue solid (gray dashed) line.

Figure IA.2. TIPS Yields with and without Carry Adjustment

problem in itself, it deserves a fuller treatment elsewhere.[4] The 5-, 7- and 10-year carry-unadjusted TIPS yields used in this analysis can be viewed as the carry-corrected TIPS yield plus a measurement error, as suggested by Equation (IA.1).

B. Parameter estimates

Table IA.1 reports all parameter estimates.

C. Mode L-I

Figure IA.3 plots results from Model L-I.

D. Decomposing Nominal Yields

Although it is not the focus of the current paper, our models can also be used to separate nominal yields into real yields, expected inflation and inflation risk premiums:

$$y_{t,\tau}^{N} = y_{t,\tau}^{R} + I_{t,\tau} + \wp_{t,\tau}. \tag{IA.2}$$

Figure IA.4 plots 1- and 10-year nominal yields and their constituents, whereas Table IA.2 reports the variance decomposition results.

These results indicate that, at least during our sample period, real yield changes explain more than three quarters of the variations in nominal yields at all maturities. Inflation expectation explains about 20% (10%) of the variations in the 1-quarter (10-year) nominal yield. Inflation risk premiums account for the remaining 2-10% of the nominal yield changes. This stands in contrast to previous studies using a longer sample period but not using TIPS yields, which typically find relatively smooth real yields but volatile inflation expectation or inflation risk premiums.[5] The limited evidence we have so far from TIPS seems to suggest that real yields may also vary considerably over time.

[4]Taking a proper account of the seasonality and carry effects is important to TIPS traders, but here in this paper we are concerned with more basic questions.

[5]See (Ang, Bekaert, and Wei, 2008, Figure 2) and (Chernov and Mueller, 2012, Figure 7) for example.

Table IA.1. Parameter Estimates

	Model NL		Model L-I		Model L-II	
State Variables Dynamics						
$dx_t = \mathcal{K}(\mu - x_t)dt + \Sigma dB_t$						
$\mathcal{K}_{11}$	0.8604	(0.9710)	0.8227	(0.3514)	0.8106	(0.8240)
$\mathcal{K}_{22}$	0.1316	(0.0534)	0.1318	(0.0536)	0.1311	(0.0549)
$\mathcal{K}_{33}$	1.4528	(1.1180)	1.4652	(0.3786)	1.4833	(0.9599)
$100 \times \Sigma_{21}$	-0.7358	(1.0533)	-0.7685	(0.5582)	-0.7724	(1.0446)
$100 \times \Sigma_{31}$	-4.4381	(16.7353)	-4.0872	(4.2439)	-3.8970	(11.1829)
$100 \times \Sigma_{32}$	-0.9482	(0.2934)	-0.9848	(0.2389)	-0.9827	(0.2644)
Nominal Pricing Kernel						
$dM_t^N/M_t^N = -r^N(x_t)dt - \lambda(x_t)'dB_t$,						
$r^N(x_t) = \rho_0^N + \rho_1^{N'} x_t,\ \ \lambda(x_t) = \lambda_0^N + \Lambda^N x_t$						
ρ_0^N	0.0472	(0.0050)	0.0471	(0.0048)	0.0471	(0.0051)
$\rho_{1,1}^N$	3.6489	(10.7748)	3.3723	(2.6618)	3.2239	(6.6502)
$\rho_{1,2}^N$	0.8774	(0.1444)	0.8813	(0.1311)	0.8763	(0.1353)
$\rho_{1,3}^N$	0.7176	(0.0607)	0.6953	(0.0294)	0.6887	(0.0407)
$\lambda_{0,1}^N$	0.3208	(0.3630)	0.3242	(0.1450)	0.3204	(0.3287)
$\lambda_{0,2}^N$	-0.4270	(0.1807)	-0.4205	(0.1859)	-0.4235	(0.1862)
$\lambda_{0,3}^N$	-1.2439	(0.3678)	-1.2907	(0.3887)	-1.3038	(0.3919)
$[\Sigma\Lambda^N]_{11}$	-0.6947	(2.8252)	-0.5810	(0.7503)	-0.5480	(1.8665)
$[\Sigma\Lambda^N]_{21}$	2.1261	(8.6766)	1.8473	(2.2209)	1.7480	(5.5985)
$[\Sigma\Lambda^N]_{31}$	3.0772	(22.3470)	2.3815	(4.8890)	2.1495	(12.1671)
$[\Sigma\Lambda^N]_{12}$	0.0329	(0.0952)	0.0340	(0.0423)	0.0322	(0.0858)
$[\Sigma\Lambda^N]_{22}$	-0.1468	(0.0344)	-0.1487	(0.0230)	-0.1485	(0.0345)
$[\Sigma\Lambda^N]_{32}$	-0.3630	(1.0038)	-0.3656	(0.2790)	-0.3533	(0.7220)
$[\Sigma\Lambda^N]_{13}$	-0.0788	(0.3092)	-0.0626	(0.1065)	-0.0577	(0.2791)
$[\Sigma\Lambda^N]_{23}$	0.5983	(0.2735)	0.5737	(0.2054)	0.5737	(0.2220)
$[\Sigma\Lambda^N]_{33}$	0.1524	(2.9980)	0.0417	(0.7189)	0.0011	(2.0248)
Log Price Level						
$d\log Q_t = \pi(x_t)dt + \sigma_q' dB_t + \sigma_q^{\perp} dB_t^{\perp},\ \pi(x_t) = \rho_0^{\pi} + \rho_1^{\pi'} x_t$						
ρ_0^{π}	0.0269	(0.0016)	0.0355	(0.0031)	0.0297	(0.0031)
$\rho_{1,1}^{\pi}$	-0.0209	(1.7085)	1.0897	(0.5285)	0.4419	(1.2812)
$\rho_{1,2}^{\pi}$	0.1072	(0.0607)	0.3698	(0.0806)	0.1952	(0.1091)
$\rho_{1,3}^{\pi}$	-0.2214	(0.4512)	0.0064	(0.1497)	0.0569	(0.3339)
$100 \times \sigma_{q,1}$	-0.0712	(0.0451)	-0.1033	(0.0411)	-0.0923	(0.0559)
$100 \times \sigma_{q,2}$	-0.0038	(0.0709)	0.0627	(0.0613)	0.0933	(0.0759)
$100 \times \sigma_{q,3}$	-0.0601	(0.0672)	0.0167	(0.0610)	-0.0100	(0.0688)
$100 \times \sigma_q^{\perp}$	0.9136	(0.0280)	0.9209	(0.0295)	0.8916	(0.0285)

Table IA.1 Continued

	Model NL		Model L-I		Model L-II	
TIPS Liquidity Premium						
$l_t = \tilde{\gamma}\tilde{x}_t + \gamma' x_t,\ d\tilde{x}_t = \tilde{\kappa}(\tilde{\mu} - \tilde{x}_t)dt + \tilde{\sigma}dW_t,\ \tilde{\lambda}_t = \tilde{\lambda}_0 + \tilde{\lambda}_1\tilde{x}_t.$						
$\tilde{\gamma}$			0.9500	(0.0287)	0.9650	(0.0292)
$\tilde{\kappa}$			0.2199	(0.2227)	0.2211	(0.2294)
$\tilde{\mu}$			0.0119	(0.0102)	0.0042	(0.0111)
$\tilde{\lambda}_0$			0.1973	(0.4285)	0.1278	(0.2954)
$\tilde{\sigma}\tilde{\lambda}_1$			-0.0876	(0.2228)	-0.0867	(0.2296)
γ_1					-0.3621	(1.1991)
γ_2					-0.1987	(0.1226)
γ_3					0.1671	(0.4911)
Measurement Errors: Nominal Yields						
$100 \times \delta_{N,3m}$	0.1314	(0.0020)	0.1312	(0.0021)	0.1312	(0.0021)
$100 \times \delta_{N,6m}$	0.0187	(0.0015)	0.0218	(0.0014)	-0.0227	(0.0014)
$100 \times \delta_{N,1y}$	0.0654	(0.0022)	0.0653	(0.0022)	0.0653	(0.0022)
$100 \times \delta_{N,2y}$	0.0000	(86.3178)	0.0000	(76.4372)	0.0000	(2232.0117)
$100 \times \delta_{N,4y}$	0.0395	(0.0016)	0.0397	(0.0016)	0.0397	(0.0016)
$100 \times \delta_{N,7y}$	-0.0000	(236.7111)	0.0000	(60.3237)	0.0000	(38.5692)
$100 \times \delta_{N,10y}$	0.0531	(0.0016)	0.0530	(0.0015)	0.0530	(0.0015)
Measurement Errors: TIPS Yields						
$100 \times \delta_{\mathcal{T},5y}$	0.6006	(0.1104)	0.0900	(0.0042)	0.0892	(0.0042)
$100 \times \delta_{\mathcal{T},7y}$	0.4707	(0.1263)	-0.0000	(1905.3160)	0.0000	(113.6434)
$100 \times \delta_{\mathcal{T},10y}$	0.4112	(0.0742)	0.0683	(0.0037)	-0.0680	(0.0037)
Measurement Errors: Survey Forecasts of Nominal Short Rate						
$100 \times \delta_{f,6m}$	0.1900	(0.0147)	0.1901	(0.0149)	0.1902	(0.0148)
$100 \times \delta_{f,12m}$	0.2979	(0.0224)	0.2980	(0.0228)	0.2981	(0.0227)

This table reports parameter estimates and standard errors for all four models we estimate. Standard errors are calculated using the BHHH formula and are reported in parentheses.

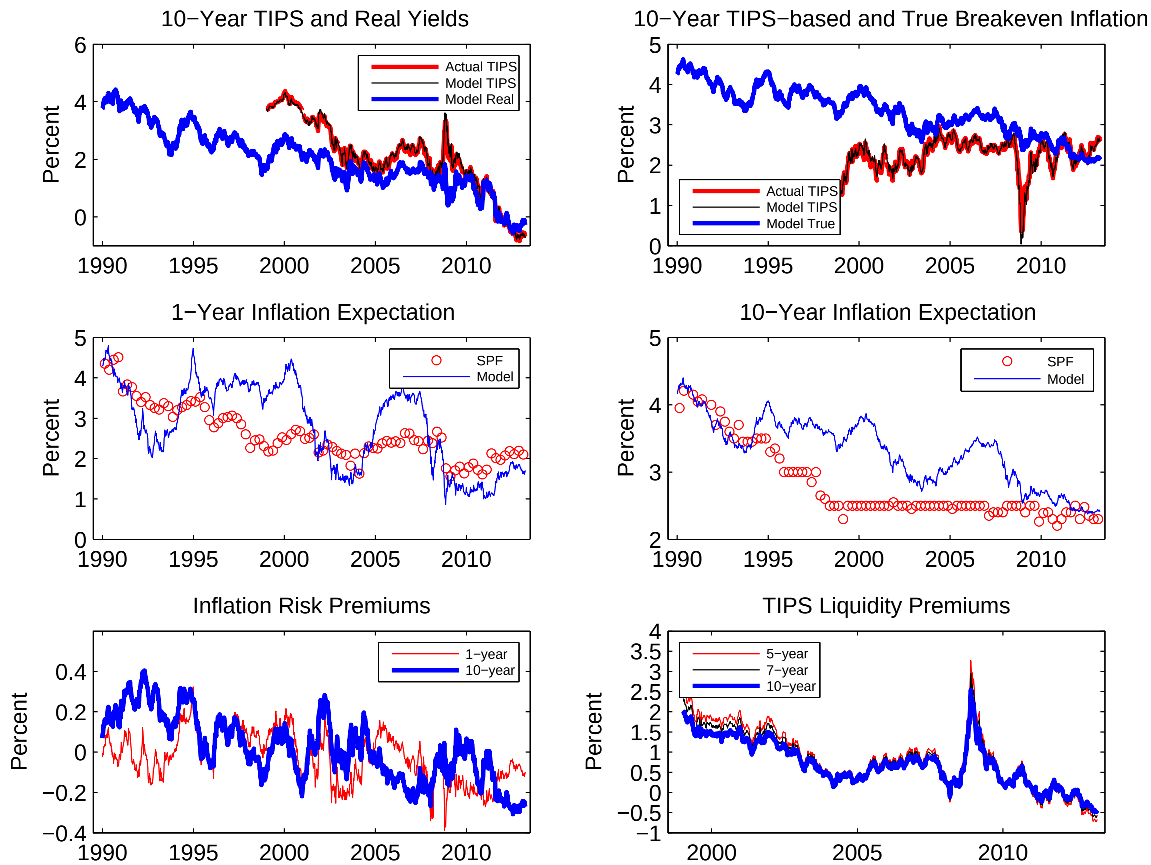


Figure IA.3. Model L-I

The top left panel plot the 10-year actual TIPS yields (red), the 10-year model-implied TIPS yields (black) and the 10-year model-implied real yields (blue). The top right panel plots the 10-year actual TIPS breakevens (red), the 10-year model-implied TIPS breakevens (black) and the 10-year model-implied true breakevens (blue). The middle panels plot the 1- and 10-year model-implied inflation expectation, respectively, together with their survey counterparts from the SPF. The bottom left panel plot the 1- and 10-year model-implied inflation risk premiums. The bottom right panel plot the fitting errors on 5-, 7-, and 10-year TIPS yields.

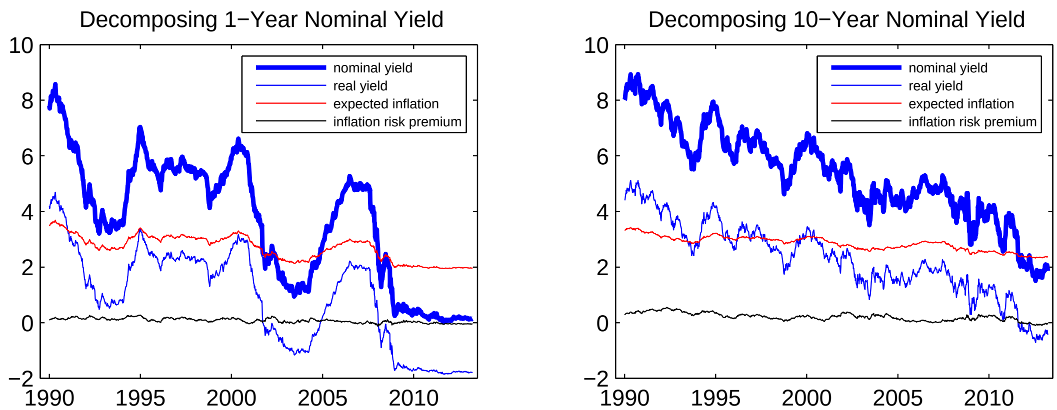


Figure IA.4. Decomposing Nominal Yields

The two panels decompose 1- and 10-year nominal yields into real yields, expected inflation and inflation risk premiums according to Equation (IA.2).

Table IA.2. In-Sample Variance decomposition of Nominal Yields

Maturity	real yield	inf exp	inf risk prem
1-quarter	0.7639	0.2214	0.0147
	(0.1078)	(0.1039)	(0.0193)
1-year	0.7743	0.2032	0.0224
	(0.1101)	(0.0987)	(0.0246)
5-year	0.7852	0.1716	0.0433
	(0.1262)	(0.0943)	(0.0579)
10-year	0.7850	0.1488	0.0663
	(0.1326)	(0.0884)	(0.0720)

Note: This table reports the in-sample variance decompositions of nominal yields into real yields, expected inflation, the inflation risk premiums, all based on Model L-II estimates. The variance decomposition is calculated according to

$$1 = \frac{cov\left(y_{t,\tau}^{N}, y_{t,\tau}^{R}\right)}{var\left(y_{t,\tau}^{N}\right)} + \frac{cov\left(y_{t,\tau}^{N}, I_{t,\tau}\right)}{var\left(y_{t,\tau}^{N}\right)} + \frac{cov\left(y_{t,\tau}^{N}, \wp_{t,\tau}^{I}\right)}{var\left(y_{t,\tau}^{N}\right)}.$$

Standard errors calculated using the delta method are reported in parentheses.

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